BACCALAUREAT GENERAL

Ecole Internationale PACA
à Manosque
Tle ES

May 3rd 2013

mathématiques OIB

- série ES –

Durée de l'épreuve : 3 heures

Coefficient : 5

Les calculatrices électroniques de poche sont autorisées.

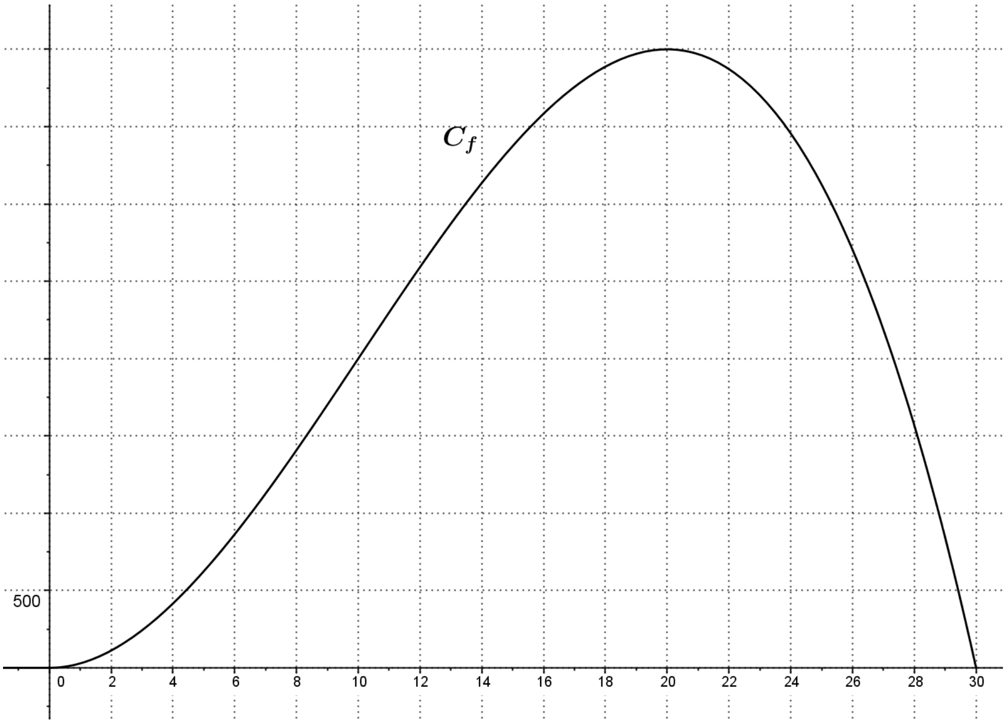
Le candidat doit traiter ces quatre exercices

Exercice 1. (5 pts)

Spreading of a disease

Part A

The graph below represents the number of sick people with respect to time t, expressed in days from the beginning of the epidemic.



1. Give the days where there are more than 2000 sick people.
2. Give the day where the number of sick people is maximal and give this maximum.
3. Estimate the day where the speed of spread of the disease is at its highest.

Part B

The number of sick people can be modelized by the function defined on $[0;30]$ by :

$f(t) = -t^3 + 30t^2$.

The spreading speed at time t is considered to be $f'(t)$.

1. Study the variations of f .
2. Sketch its table of variation and check the result found in part A question 2.
3. Calculate the second derivative $f''(t)$.
4. a. Study the variations of f'.

b. Deduce the convexity of f and provide an interpretation of it.

c. Prove that the graph has an inflexion point and give its practical meaning.

d. Calculate the spreading speed on the 10th day. Give an interpretation of this value.

Exercise 2. (6 pts) Baccalauréat ES Polynésie septembre 2012

A company produces chemicals. It can produces x cubic-meters every day, x being in the interval $[1;6]$.

The total production cost $C_T(x)$ (expressed in thousands of €) is given by :

$C_T(x) = \dfrac{x^2}{2} + 4\ln x + 5.6$

1. Prove that this function C_T is strictly increasing on $[1;6]$.
2. We denote $C_M(x)$ the average production cost for a daily production of $x\,m^3$ and recall that $C_M(x) = \dfrac{C_T(x)}{x}$.
   a. Express $C_M(x)$ in terms of x.
   b. We take as given that the function C_M is differentiable on $[1;6]$. Calculate $C'_M(x)$ and prove that $C'_M(x) = \dfrac{x^2 - 3.2 - 8\ln x}{2x^2}$ for any x in $[1;6]$.
3. Consider the function defined on $[1;6]$ by $f(x) = x^2 - 3.2 - 8\ln x$.
   a. We take as given that the function f is differentiable on $[1;6]$. Study it's variations.
   b. Prove that the equation $f(x) = 0$ has a unique solution α in $[2;6]$ and give an approximate value of α (rounded to 0.1)
   c. Deduce the sign of f on $[1;6]$.
4. Take for α the value found in 3b.
   a. Using the results of question 3, study the variations of C_M on $[1;6]$. Give its table of variations (with the values rounded to 0.1)
   b. What is the minimum average cost ?
5. a. Check that the function defined on $[1;6]$ by $x \mapsto x\ln x - x$ is an antiderivative of $x \mapsto \ln x$ on this interval.
   c. Deduce an antiderivative of C_T on $[1;6]$.
   d. Calculate the average total cost if the company produces between 1 and 6 cubic-meters every day.

Exercice 3. (5 pts)

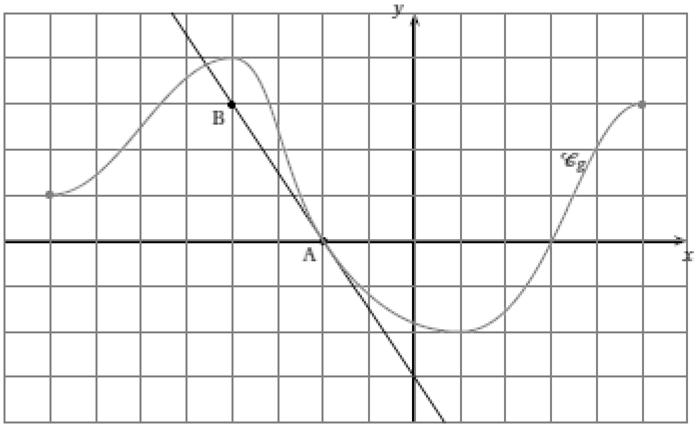
A company produces a very large amount of electronic components (and especially hard-drives) for the computer industry. 40% of the components are hard-drives. We take 60 components randomly out of the stock (which is considered big enough to consider this sampling as with replacement). We consider the random variable X which is the number of hard-drives in such a sample.

1. a. What is the probability distribution of the variable X ? give its parameters.
   b. Using your calculator, calculate $P(20 \le X \le 28)$.
2. We now assume $X \sim \mathcal{N}(24;14.4)$. Calculate $P(20 \le X \le 28)$.
3. a. Prove that $P(24 - a \le X \le 24 + a) = 1 - 2 \times P(X \le 24 - a)$.
   b. Deduce that finding the number **a** such that $P(24 - a \le X \le 24 + a) \approx 0.90$ comes down to find the number **a** such that $P(X \le 24 - a) \approx 0.05$.
   c. Using the table opposite, find the minimal number of hard-drives a such that at least 90% of the samples contain between $24 - a$ and $24 + a$ hard-drives.

x	$P(X \le x)$
16	0.0175
17	0.0325
18	0.0569
19	0.0938

Exercice 4. (4 pts) Baccalauréat ES Pondichéry avril 2012

This exercise is multiple choice. Only one answer is correct. No justifications are required.
Just write down on your paper the number of the question and the letter of your answer.
A correct answer gives 1 point. Missing or incorrect answers don't take any points off.



1. Opposite is the graph of a function g defined and differentiable on $[-8;5]$. The line (AB) drawn is the tangent to this curve at point A with x-coordinate -2. We denote g' the derivative of g on $[-8;5]$.
   a. $g'(-2)=-1.5$
   b. $g'(-2)=0$
   c. $g'(-2)=-\dfrac{2}{3}$

2. Let $I=\int_2^7\left(2x+1-\dfrac{1}{x}\right)dx$;
   a. $I=50+ln\left(\dfrac{2}{7}\right)$
   b. $I=48.7$
   c. $I=10-\dfrac{1}{7}+\dfrac{1}{2}$

3. Let f be the function defined on ℝ by $f(x)=\dfrac{6x-5}{3x^2-5x+7}$. We denote F the primitive of f such that $F(1)=1$.
   a. For any x in ℝ : $F(x)=ln(3x^2-5x+7)$
   b. For any x in ℝ : $F(x)=ln(3x^2-5x+7)+1-ln5$
   c. For any x in ℝ : $F(x)=\dfrac{1}{3x^2-5x+7}+1$

4. $\dfrac{ln(e^2)}{ln16}$ is equal to :
   a. $2ln\left(\dfrac{e}{4}\right)$
   b. $\dfrac{1}{2ln2}$
   c. $2lne-ln16$

Correction available on Chamilo

Exercice 3. (4 pts) BAC ES Polynésie septembre 2011

Exercice 4. (5 pts) Baccalauréat ES Pondichéry avril 2012

1. $g'(-2) = -1.5$
2. $I = 50 + ln\left(\frac{2}{7}\right)$
3. $F(x) = ln(3x^2 - 5x + 7) + 1 - ln5$
4. $\frac{1}{2ln2}$