Continuity
(FR : continuité)

1. Continuous function

Intuitive notion of continuity :
Consider f a function defined on an interval I . We say that f is continuous on I if its graph doesn't have any gap, that is, if we can draw it "without lifting the pen"

Notes :

- ✓ Counter example : E(x), the Ceiling function (fr : function partie entière)
- ✓ <u>The arrows in a table of variation indicate both the continuity and the strict increase (or decrease) of the function on the corresponding intervals.</u>

Properties :

- ***If a function f is differentiable on an interval I, then it is continuous on I. (but the inverse is false)***
- ***All the functions build (by addition, multiplication, composition) from the standard functions are continuous on their domain.***

2. Properties of the continuous functions.

2.1 Intermediate value theorem (Fr : Theoreme des valeurs intermediaires)

Theorem (taken as admitted):
Let f be a continuous function on an interval I and a and b two real numbers belonging to I.
For any real number k between $f(a)$ and $f(b)$, there exists a real number c between a and b such that : $f(c)=k$

Note :

- ✓ In other words, $f(x)$ takes any intermediate value between $f(a)$ and $f(b)$.
- ✓ This theorem is often used to prove that a given equation (we can't solve algebraically for eg) has at least one solution.

Example : $2x^3+3x^2-2=0$

- ✓ The calculator helps finding an approximate value of the solutions (function « zéros » or « root » or by browsing the table of values)

IMPORTANT SPECIAL CASE OF THE IVT

Property :
Let f be a continuous strictly monotonic function on an interval $[a;b]$. For any real number k $k \in [f(a);f(b)]$, the equation $f(x)=k$ has a <u>unique</u> solution in $[a;b]$.

Note :

- ✓ This property remains valid for other types of intervals, bounded or not.
- ✓ Be careful to well check all the conditions are met to apply the IVT

Example :

Find the number of solutions of the equation $x^3-3x^2=-5$ and give a rounded value to 0.01

Key points of the chapter

- ✓ Knowing what is a continuous function and recognize them
- ✓ Applying the IVT
- ✓ Being able to use ICT to find roots of functions