Probability (part I) :
Conditional probability

1. SHORT RECAP.

Key words- Random experiment / *Expérience aléatoire* — Outcome / *Issue* — Event / *Evénement* — Sample space / *Univers*

Example

Experiment: Pick a card from a standard pack of 52 cards
Outcomes: 2 of hearts, 9 of clubs, ace of spades, etc.
Sample space: set of all the cards.
Event: picking a card with an even number on.

Bottom-line definitions and properties
Let A and B be two events.

An event is a set of possible outcomes. P("even number") = $\frac{20}{52}=\frac{5}{13}$

$0 \le P(A) \le 1$ $\qquad P(\bar{A})=1-P(A)$

If A and B are mutually exclusive (so that they cannot occur at the same time, fr: incompatibles), then $P(A \text{ or } B)=P(A\cup B)=P(A)+P(B)$

More generally, if A and B are not mutually exclusive:

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

2. CONDITIONAL PROBABILITY. (FR : *PROBABILITE CONDITIONNELLE*)

Introductory example: The probability that it will be sunny tomorrow is $\frac{1}{3}$. If it is sunny, the probability that Susan plays tennis is $\frac{4}{5}$. If it is not sunny, the probability that Susan plays tennis is $\frac{2}{5}$.
What is the probability that Susan plays tennis ?

P(tennis) = P(sunny AND tennis) + P(not sunny AND tennis)

$$=\frac{1}{3}\times\frac{4}{5}+\frac{2}{3}\times\frac{2}{5}=\frac{8}{15}$$

Definition :
Let's consider two events A and B such that $P(A)\neq 0$. ***The probability of B given A (B sachant A), denoted*** $P_A(B)$ ***(or*** $P(B|A)$ ***in Britain) is defined by :*** $P_A(B)=\frac{P(A\cap B)}{P(A)}$

Notes :

- the previous formula can be written $\boxed{P(A\cap B)=P(A)\times P_A(B)}$ (called the Bayes' formula) to work out $P(A\cap B)$ from $P_A(B)$ which happens quite often.
- Using a tree diagram : **THE RULES**
  - the weighted branches from second level onwards are representing conditional probabilities
  - the total value of the probabilities issued from one node is 1 (rule of nodes).
  - the probability of a leaf (outcome) on the tree is obtained by multiplying the values on the branches to get to it.

Example 1 : In the situation above

3. LAW OF TOTAL PROBABILITY.

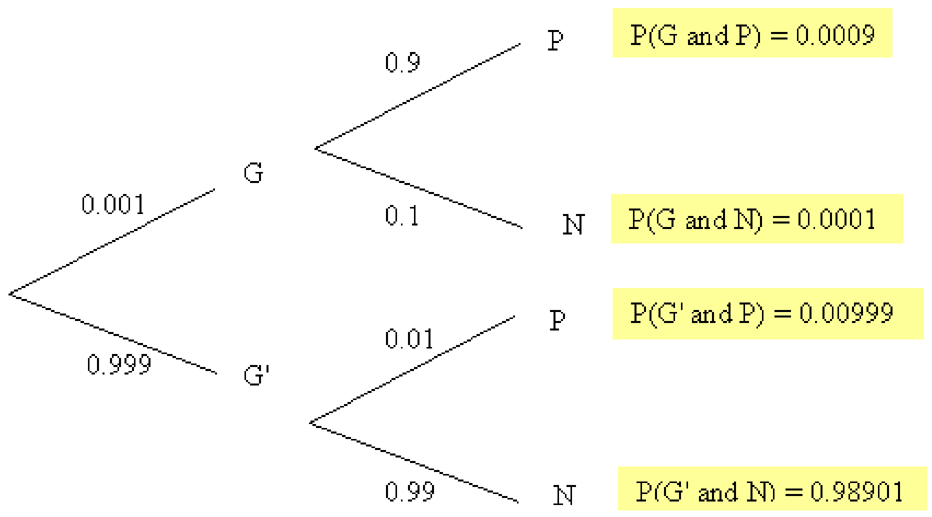
Definition :
A set of events $B_1, B_2, ...B_n$ are forming a partition of the sample space U if they are mutually exclusive to each other and if their union is U.

Law of total probability : (formule des probabilities totales)
$B_1, B_2, ...B_n$ ***forming a partition of a sample space U, for any event E, we have :***
$$P(E)=P(B_1)\times P_{B_1}(E)+P(B_2)\times P_{B_2}(E)+......P(B_n)\times P_{B_n}(E)$$

Example 2: Detecting a disease

0.1% of the population carry a particular faulty gene. A test exists for detecting whether an individual is a carrier of the gene. In people who actually carry the gene, the test provides a positive result with probability 0.9. In people who don't carry the gene, the test provides a positive result with probability 0.01.
If someone gives a positive result when tested, find the probability that they actually are a carrier of the gene.

Let G = person carries gene P = test is positive for gene N = test is negative for gene
The tree diagram then looks as follows:



We want to find $P(G \mid P) = \dfrac{P(G \text{ and } P)}{P(P)}$

However, P(P) = P(G and P) + P(G' and P) = 0.0009 + 0.00999 = 0.01089

Therefore, $P(G \mid P) = \dfrac{0.0009}{0.01089} = 0.0826$ (to 3 significant figures)

So there is a very low chance of actually having the gene even if the test says that you have it.

Note: This example highlights the difficulty of detecting rare conditions or diseases.

Key points of the chapter

- ✓ Drawing up a tree diagram associated with a given problem
- ✓ Using it to find out probabilities
- ✓ Using the total probabilities law
- ✓ Performing probabilities simulations using ICT