TLE ES HOMEWORK
TO HAND IN BY OCTOBER 26TH 2012

Ex1 : A business and economics problem around marginals

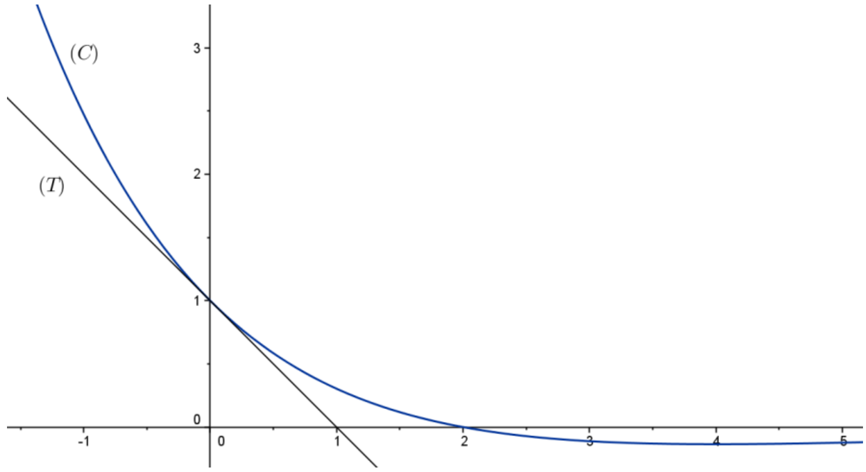
A widget manufacturer has determined that :

- the demand function for his widgets is $p(x)=\dfrac{1000}{\sqrt{x}}$. That means the market will ask for x widgets if the selling price is p(x).
- the cost of producing x widgets is given by the following cost function : $C(x)=10x+100\sqrt{x}+10\,000$. (10.000 are the fixed costs and $10x+100\sqrt{x}$ the variable costs)

1. Determine the marginal cost at $x=100$. Make a sentence to give an interpretation of this value.
2. Work out the unit cost (or average cost) $UC(x)$ (cost of one widget if he produces x of them)
3. a) Work out the revenue $R(x)$ the manufacturer will get (selling x widgets)
   b) Determine the marginal revenue at $x=100$. Make a sentence to give an interpretation of this value.
4. a) Work out the profit $P(x)$ the manufacturer will get (selling x widgets)
   b) Determine the marginal profit at $x=100$. Make a sentence to give an interpretation of this value.
5. How many widgets should be manufactured and what should they be sold for to produce the maximum profit ?
6. Draw up the graphs of $C(x)$ and $R(x)$ and make appear on it the previous answer. (you may draw it by hand or using a graphing software, take 1cm for 250 widgets on the x-axis and 1cm for 2000€ on the y-axis)

Ex2 : Using a graph and a CAS Software

Let's consider the function f defined on $\mathbb{R}$ by : $f(x)=(ax+b)e^{-0.5x}$ where a and b are fixed real numbers to be worked out. Opposite is the graph (C) of the function f and it's tangent (T) at $A(0;1)$.



1. a) using the graph, find the values of $f(0)$ and $f'(0)$.
   b) We got $f'(x)$ thanks to a CAS software (here "XCAS en ligne") :

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f(x):=(a*x+b)*exp(-0.5x)
                                        (x)->(a*x+b)*exp(-0.5*x)
deriver(f(x))
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$$ae^{-(0.5x)}+(ax+b)e^{-(0.5x)}*-0.5$$

   Admitting this result, justify that the real numbers a and b are the solutions of the simultaneous equations $\begin{cases} b=1 \\ -a+0.5b=1 \end{cases}$

   Bonus : find yourself $f'(x)$ by calculation

   c) deduce that $f(x)=(-0.5x+1)e^{-0.5x}$
2. a) Analyse the sign of $f'(x)$. Deduce the table of variation of f.
   b) What is the minimum of f on $\mathbb{R}$? For which value is it achieved ?
   Check with a graphing software (Geogebra for eg) (give a screen copy)
3. Work out the equation of (T).

T^{LE} ES DM

CORRECTION

Lycée International Manosque

1. The marginal cost at 100 is roughly $C'(100)$. As $C'(x)=10+\dfrac{100}{2\sqrt{x}}=10+\dfrac{50}{\sqrt{x}}$, we have $C'(100)=10+\dfrac{50}{10}=15$. It means that the 101th widget cost about 15€ to produce.
2. The unit cost (or average cost) is $UC(x)=\dfrac{C(x)}{x}=10+\dfrac{100\sqrt{x}}{x}+\dfrac{10000}{x}$
3. a) The revenue $R(x)$ the manufacturer will get (selling x widgets) is $R(x)=x\times p\left(x\right)$ (as he sells x widgets at p(x)€ each). So $R(x)=\dfrac{1000x}{\sqrt{x}}=1000\sqrt{x}$

b) The marginal revenue at 100 is roughly $R'(100)$. As $R'(x)=\dfrac{1000}{2\sqrt{x}}=\dfrac{500}{\sqrt{x}}$, we have $R'(100)=\dfrac{500}{10}=50$. It means that the revenue due to the 101th widget is about 50€.

4. a) The profit $P(x)$ the manufacturer will get (selling x widgets) is $P(x)=\underbrace{R\left(x\right)}_{\text{earnings}}-\underbrace{C(x)}_{\text{expenses}}$. So

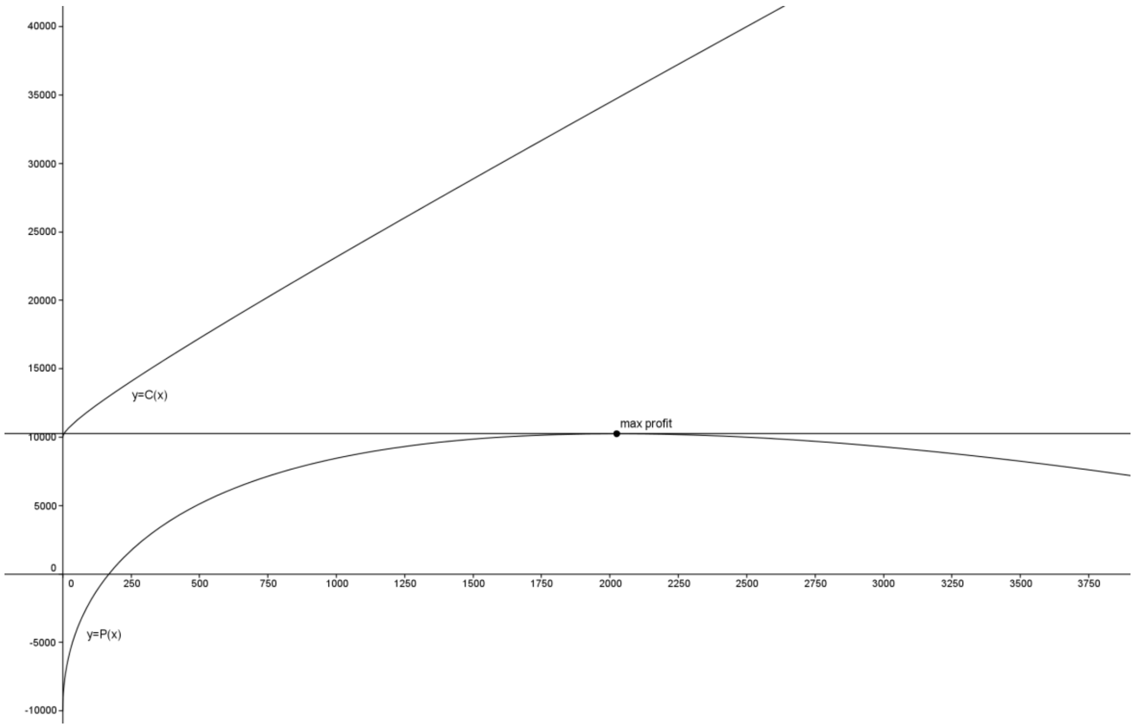
$$P(x)=\frac{1000x}{\sqrt{x}}-\left(10x+100\sqrt{x}+10000\right)=\ldots=900\sqrt{x}-10x-10000$$

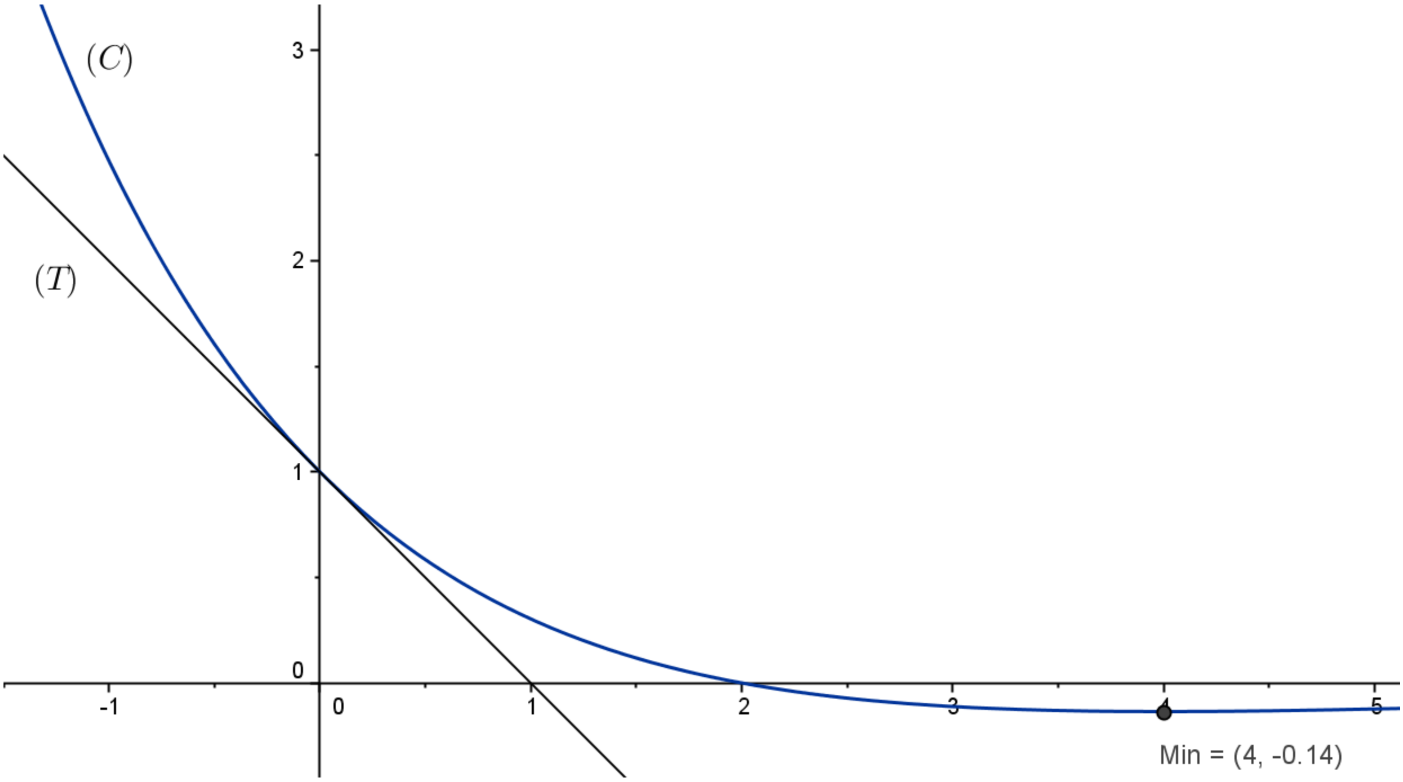
b) The marginal profit at 100 is roughly $P'(100)$. As $P'(x)=\dfrac{900}{2\sqrt{x}}-10=\dfrac{450}{\sqrt{x}}-10$, we have $P'(x)=\dfrac{450}{10}-10=35$. (we also could have done $P'(x)=R'(x)-C'(x)$). It means that the profit due to the 101th widget is about 35€.

5. The maximum profit will be obtained while the derivative $P'(x)$ is null and changes its sign.

$P'(x)=0\Leftrightarrow\dfrac{450}{\sqrt{x}}-10=0\Leftrightarrow\dfrac{450}{\sqrt{x}}=10\Leftrightarrow\sqrt{x}=45\Leftrightarrow x=45^2=2025$. The sign changes from positive to negative. The maximum profit will be for 2025 widgets sold. The profit is then 10.250€ and the selling price 22.22€.

6.





(C)
(T)
3
2
1
0
-1
0
1
2
3
4
5
Min = (4, -0.14)