Tle ES **DS n°1**

SEPTEMBER 21ST 2012

duration : 45'

EXERCICE 1. (3 PTS)

Analyse the sign of the following quadratic function : $P(x) = -3x^2 - 10x + 8$

EXERCICE 2. (8.5 PTS)

1. Calculate the equation of the tangent to the curve $y = x^3 - 2x^2 + 5$ at 2
2. Find the derivatives of the following functions:

   $g(x) = x + 3 + \frac{1}{x}$ $\quad h(x) = \frac{x^2+2}{1-x}$ $\quad i(x) = \frac{5}{3x^2 - 2x + 4}$

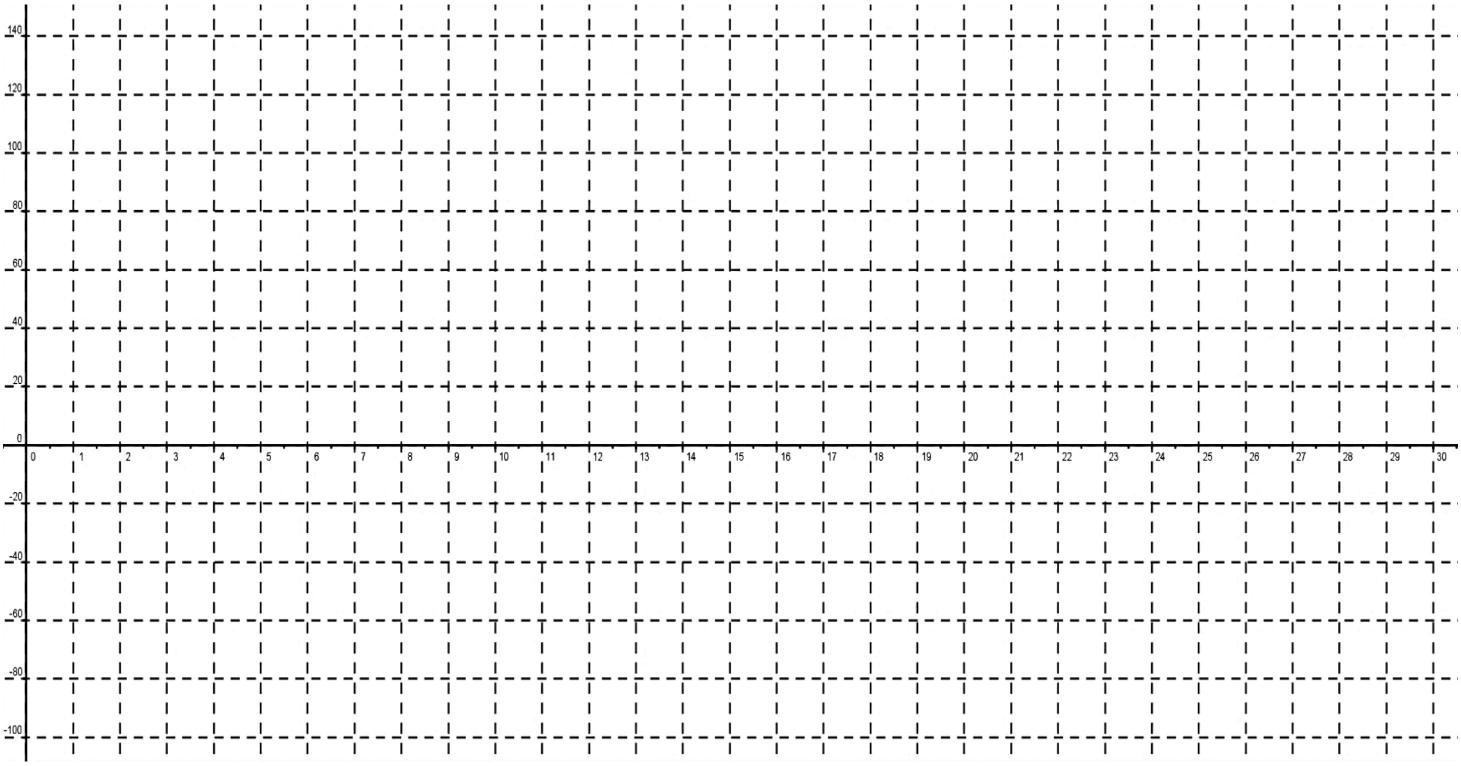
   $k : x \mapsto (x^3 - 1)(3x^2 + x)$
3. How has to be a derivative to allow you to do a sign study ?

EXERCICE 3 (8.5 PTS).

The total cost of production, expressed in Euros, of x objects is $C(x) = x^2 - 10x + 96$, for $x \in [1;30]$

Each object is sold for 20 euros.

1) Prove that the profit function is $P(x) = -x^2 + 30x - 96$.
2) Calculate the derivative of P.
3) Draw up the table of variations of P.
4) Deduce the quantity of objects sold which maximizes profit. What is this profit ?
5) Give the equation of the tangent of the graph of P at 4.
6) Sketch the graph of P and its tangent at 4.



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CORRECTION

Exercice 1. (3 pts)

$\Delta = b^2 - 4ac = 100 + 4\times 3\times 8 = 196 = 14^2$ So the quadratic function has two roots : $x_1 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{10-14}{-6} = \frac{2}{3}$ and $x_2 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{10+14}{-6} = -4$.

Its sign is the same than a (here -3) outside the roots (parabola downwards), so it is positive on $\left[-4;\frac{2}{3}\right]$ and negative on $\left]-\infty;-4\right]\cup\left[\frac{2}{3};+\infty\right[$.

Exercice 2. (8.5 pts)

1. the tangent formula $y = f'(a)(x-a)+f(a)$ gives here $y = f'(2)(x-2)+f(2)$ with $f'(x) = 3x^2-4x$ so $f'(2)=4$ and $f(2)=5$. Finally: $y = 4(x-2)+5 = 4x-3$

2.

$$g'(x) = 1-\frac{1}{x^2}$$

$$h'(x) = \frac{u'v-uv'}{v^2} = \frac{2x(1-x)-\left(x^2+2\right)(-1)}{(1-x)^2} = \frac{-x^2+2x+2}{(1-x)^2}$$

$$i'(x) = -\frac{v'}{v^2} = 5\frac{6x-2}{\left(3x^2-2x+4\right)^2} = \frac{10(3x-1)}{\left(3x^2-2x+4\right)^2}$$

$$k'(x) = u'v+uv' = \left(3x^2\right)\left(3x^2+x\right)+\left(x^3-1\right)(6x+1) = \ldots = 15x^4+4x^3-6x-1$$

3. It has to be factorised.

Exercice 3 (8.5 pts).

The total cost of production, expressed in Euros, of x objects is $C(x) = x^2-10x+96$, for $x\in[1;30]$

Each object is sold for 20 euros.

1) Since each object is sold for 20 euros, the income is $20x$, so the profit function is $P(x) = 20x-\left(x^2-10x+96\right) = -x^2+30x-96$.

2) $P'(x) = -2x+30$

3) .

x	1		15		30
$P'(x)$		+	0	−	
P		↗	129	↘	

4) The quantity of objects sold which maximizes profit is 15. The profit is then 129€.

5) The tangent formula $y = f'(a)(x-a)+f(a)$ gives here $y = f'(4)(x-4)+f(4)$ with $f'(4)=22$ and $f(4)=8$. Finally: $y = 22(x-4)+8 = 22x-80$

6) .