TLE ES TEST N°2

FRIDAY OCTOBER 12TH 2012

duration : 50'

EXERCICE 1. (5 PTS)

Max is renting a flat for a monthly rent of 450€. This rent increases by 2% yearly. For any integer number n, we denote u_n the value of his rent after n increases.

1. What kind of sequence is (u_n) ? Express u_n in terms of n.
2. What is it's sense of variation ? What is it's limit ?
3. After how many years will the rent exceed 500€ ?
4. Max will stay 5 years in his flat. What will be the total amount of money he will have spent for renting this flat ?

EXERCICE 2. (4 PTS)

"***You just won a fabulous game offering you 300 000€ each day during one month. The only counter-part is for you to give back 1 ct on the first day, 2 cts on the second day, 3 cts on the third day and so on, doubling every day the amount you have refunded the day before***"
Seeing this advertising, would you be keen on accepting the game ? Even during a February month ?

EXERCICE 3. (7 PTS) MCQ

2 wrong answers cancel a right one

1. Among the following sequences, which ones are geometric ?

   $u_n = 2\times 4^n$ $\qquad v_n = \dfrac{3}{4^{n+1}}$ $\qquad w_n = 2n$ $\qquad z_n = 3\times 2^n - 1$

2. u is a geometric sequence with u_4 and u_6. What can be the value of u_5 ?

   -3 $\qquad$ 3 $\qquad$ -12 $\qquad$ 12

3. Among the following sequences, which ones are increasing ?

   $u_n = 4n+5$ $\qquad v_n = 3^n$ $\qquad w_n = 10\times 0.5^n$ $\qquad z_n = 2\times 1.01^n$

4. u is a geometric sequence with common ratio 2 and $u_0 = 3$. What can be the value of the sum $u_0 + u_1 + u_2 ..... + u_{10}$?

   2047 $\qquad$ 177 146 $\qquad$ 253 $\qquad$ 6141

5. q is a real number such that $1 + q + q^2 ..... + q^5 = 19608$. What is the value of q ?

   4 $\qquad$ 5 $\qquad$ 6 $\qquad$ 7

6. Among the following sequences, which ones have a finite limit ?

   $u_n = \left(\dfrac{5}{2}\right)^n$ $\qquad v_n = 0.5^n - 2^n$ $\qquad w_n = 2 - \left(\dfrac{1}{10}\right)^n$ $\qquad z_n = 3\times 0.5^n$

EXERCICE 4. (4 PTS)

Calculate the derivatives of the following functions :

$f(x) = \dfrac{5x-3}{2x}$ $\qquad g(x) = (2x+1)\sqrt{x}$ $\qquad h(x) = \dfrac{-3}{2x}$ $\qquad k(x) = 3x^3 - 5x + \dfrac{2}{x}$

Exercice 1. (5 pts)

1. (u_n) is geometric since, from one year to the next, the rent is always multiplied by the same constant number : 1.02. Hence $u_n = u_0 \times 1.02^n = 450 \times 1.02^n$.
2. Since the common ration is >1 , the sequence is increasing and its limit is $+\infty$
3. The rent will exceed 500€ after 6 years.

0	450
1	459
2	468,18
3	477,5436
4	487,094472
5	496,836361
6	506,773089

4. The total amount of money he will have spent for renting this flat is $12 \times (u_0 + u_1 + ... + u_4) = 12 \times \dfrac{1.02^5 - 1}{1.02 - 1} = 28102€$

Exercice 2. (4 pts)

		amount you win	amount you give back		
28	days	8 400 000 €	2 684 355 €	$0.01 \times \dfrac{2^{nb\ de\ jours} - 1}{2-1}$	
30	days	9 000 000 €	10 737 418 €		
31	days	9 300 000 €	21 474 836 €		

Exercice 3. (7 pts) MCQ

1. Among the following sequences, which ones are geometric ?

   $\boxed{u_n = 2 \times 4^n}$ $\qquad$ $\boxed{v_n = \dfrac{3}{4^{n+1}}}$ $\qquad$ $w_n = 2n$ $\qquad$ $z_n = 3 \times 2^n - 1$

2. u is a geometric sequence with u_4 and u_6. What can be the value of u_5 ?

   -3 $\qquad$ 3 $\qquad$ $\boxed{-12}$ $\qquad$ $\boxed{12}$

3. Among the following sequences, which ones are increasing ?

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4. u is a geometric sequence with common ratio 2 and $u_0 = 3$. What can be the value of the sum $u_0 + u_1 + u_2 ..... + u_{10}$?

   2047 $\qquad$ 177 146 $\qquad$ 253 $\qquad$ $\boxed{6141}$

5. q is a real number such that $1 + q + q^2 ..... + q^5 = 19608$. What is the value of q ?

   4 $\qquad$ 5 $\qquad$ 6 $\qquad$ $\boxed{7}$

6. Among the following sequences, which ones have a finite limit ?

   $u_n = \left(\dfrac{5}{2}\right)^n$ $\qquad$ $v_n = 0.5^n - 2^n$ $\qquad$ $\boxed{w_n = 2 - \left(\dfrac{1}{10}\right)^n}$ $\qquad$ $\boxed{z_n = 3 \times 0.5^n}$

Exercice 4. (4 pts)

With the Leibniz notations : u' is denoted $\dfrac{du}{dx}$

$$f'(x) = \left(\frac{5x-3}{2x}\right)' = \left(\frac{u}{v}\right)' = ... \frac{3}{2x^2}$$ see opposite

$$g'(x) = \left[(2x+1)\sqrt{x}\right]' = [u \times v]' = ... = \frac{6x+1}{2\sqrt{x}}$$

$$h'(x) = \left(\frac{-3}{2x}\right)' = -3\left(\frac{1}{u}\right)' = \frac{3}{2x^2}$$

$$k'(x) = \left[3x^3 - 5x + \frac{2}{x}\right]' = 9x^2 - 5 - \frac{2}{x^2}$$

Possible derivation:

$\dfrac{d}{dx}\left(\dfrac{5x-3}{2x}\right)$

Factor out constants:

$\dfrac{1}{2}\left(\dfrac{d}{dx}\left(\dfrac{5x-3}{x}\right)\right)$

Use the quotient rule, $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$, where $u = 5x - 3$ and $v = x$:

$\dfrac{1}{2}\,\dfrac{x\left(\frac{d}{dx}(5x-3)\right) - (5x-3)\left(\frac{d}{dx}(x)\right)}{x^2}$

The derivative of x is 1:

$\dfrac{x\left(\frac{d}{dx}(5x-3)\right) - 1\,(5x-3)}{2x^2}$

Differentiate the sum term by term and factor out constants:

$\dfrac{x\left(5\left(\frac{d}{dx}(x)\right) + \frac{d}{dx}(-3)\right) - 5x + 3}{2x^2}$

The derivative of -3 is zero:

$\dfrac{x\left(5\left(\frac{d}{dx}(x)\right) + 0\right) - 5x + 3}{2x^2}$

The derivative of x is 1:

$\dfrac{5x - 5x + 3}{2x^2}$