TEST N°5
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Exercice 1. (11 pts) from Baccalauréat ES Antilles septembre 2011

An enterprise manufactures and sells photovoltaic solar panels. It produces between 50 and 2500 panels a month.
Let f be the function defined on $[0.5;25]$ by $f(x)=18\ln x-x^2+16x-15$.
If x represents the number (in hundreds) of produced and sold solar panels, then we assume $f(x)$ represents the enterprise's monthly benefit (in thousands €).
We assume f is differentiable on $[0.5;25]$, and we denote f' its derivative.

1. Calculate $f'(x)$. Check that, for any x in $[0.5;25]$, we have $f'(x)=\dfrac{-2x^2+16x+18}{x}$.
2. Study the sign of $f'(x)$ on $[0.5;25]$. Deduce then the variations of f on $[0.5;25]$.
3. a. Calculate $f(1)$.

   b. Prove that, on $[18;19]$, the equation $f(x)=0$ has a single solution α. Give an approximate value of α rounded to 10^{-2} .

   c. Deduce then the sign of $f(x)$ for any x in $[0.5;25]$.
4. What is the smallest and biggest number of panels the enterprise has to produce to be profitable ?
5. *In this question, any trace of research, even incomplete, will be taken into account in the mark.*
   Is it possible for the enterprise to reach a monthly benefit of 100 000 € ? Justify.

Exercice 2. (9 pts) from Baccalauréat ES Antilles septembre 2011

1. Find the smallest value of n (integer) such that :

   a. $2^n > 10\,000$ b. $3\times 0.85^n \le 0.02$
2. Calculate the derivatives of the following functions :

   a. $f(x)=x\ln x-x$ b. $f(x)=\dfrac{\ln x+1}{x}$ c. $f(t)=(\ln t+1)(\ln t-2)$
3. Solve the equation/inequation :

   a. $e^x+2=3$ b. $2\ln x+1>0$
4. Simplify the expresion :

   a. $2\ln 10-3\ln 5+\ln 2$ b. $\ln\dfrac{1}{2}-4\ln 2+\ln 16$

TEST N°5
CORRECTION

Exercice 1. (11 pts) from Baccalauréat ES Antilles septembre 2011

An enterprise manufactures and sells photovoltaic solar panels. It produces between 50 and 2500 panels a month.

1. $f'(x) = \frac{18}{x} - 2x + 16 = \frac{-2x^2 + 16x + 18}{x}$ for any x in $[0.5; 25]$.
2. The sign of $f'(x)$ on $[0.5; 25]$ is the same as $-2x^2 + 16x + 18$, since x is positive.
   $-2x^2 + 16x + 18 = 2(-x^2 + 8x + 9)$ which discriminant is $\Delta = b^2 - 4ac = 64 + 4 \times 9 = 100 = 10^2$.
   Its roots are then $x_1 = \frac{-8-10}{-2} = 9$ and $x_2 = \frac{-8+10}{-2} = -1$. Its sign is the one of a (here negative) outside theses roots, so $f'(x)$ is positive on $[-1; 9]$
   The variations of f on $[0.5; 25]$ are thus :

x	0.5		9		25
$f'(x)$		+	0	−	
f	↗		≈ 87.55		↘

3. a. $f(1) = 18\underbrace{ln1}_{=0} - 1^2 + 16 - 15 = 0$.
   b. on $[18; 19]$, f in continuous and strictly decreasing from $f(18) \approx 1 > 0$ to $f(19) \approx -19 < 0$. Hence according to the Intermediate Value theorem, the equation $f(x) = 0$ has a single solution α. An approximate value of α : 18.05 rounded to 10^{-2} .
   c. The sign of $f(x)$ for any x in $[0.5; 25]$ is thus :

x	0.5		1		α		25
Sign of f		−	0	+	0	−	

4. The enterprise has to produce between 100 and 1800 panels to be profitable.
5. The highest monthly benefit is obtained for x=9 and its value is ≈ 87550, so can't reach 100 000 €.

Exercice 2. (9 pts)

All done in class