TEST N°7
APRIL 5^{TH} 2013

EXERCISE 1 (8 PTS) MCQ

Give all the right answers

1. The density function of a random variable X is defined on $[0;1]$ by $f(x)=kx^2$ where $k\in\mathbb{R}$.
   a. $k=3$ b. $P(X=2)=4k$ c. $P(0.5\leq X\leq 1)=0.5$
2. The random variable $X\sim\mathcal{N}(0;1)$.
   a. Its density function is defined by $f(x)=e^{\frac{-x^2}{2}}$ b. $P(-1\leq X\leq 1)=0.5$
   c. $P(-1.96\leq X\leq 1.96)\approx 0.95$
3. The random variable $X\sim\mathcal{N}(20;36)$. The smallest integer n such that is
   a. $n=14$ b. $n=6$ c. $n=12$
4. The random variable associated with the age at which young children first talk follows a normal distribution with a mean value 11,5 and a standard deviation 3.2 (in months).
   a. $P(X\leq 9)\approx 0.21$ b. $P(X\geq 15)\approx 0.86$ c. $P(8\leq X\leq 12)=0.5$
5. Mike has to take his bus at 7h00. He regularly arrives late and the next one is at 7h30. The random variable associated with the late time of Mike follows a uniform distribution on $[0;30]$.
   a. The density function of X is $f(x)=30$
   b. The probability for Mike to wait between 10' and 15' is $\frac{1}{6}$
   c. Mike can expect to wait, on average, less than 10'.
6. f is the function defined on $\mathbb{R}$ by $f(x)=2e^{-x}+1$. An approximate value of the mean value of f between 0 and 1, rounded to the thousandth is :
   0.213 1.264 2.264 -0.264
7. The primitive F of the function f defined on $]0;+\infty[$ by $f(x)=\frac{2x^2-x+3}{x}$ such that $F(1)=1$ satisfies :

$$F(x)=\frac{\frac{2}{3}x^3-\frac{1}{2}x^2+3x}{\frac{1}{2}x^2}-\frac{17}{3} \qquad F(x)=x^2-1+3\ln x$$

$$F(x)=x^2-x+3\ln x+1 \qquad F(x)=2-\frac{3}{x^2}+1$$

8. f is the function defined on $]0;+\infty[$ by $f(x)=\frac{5}{x}$. The area (expressed in Units of Area) bounded by its graph $\mathcal{C}$, the x-axis and the lines with equation $x=1$ and $x=2$ is equal to :
   $\ln\frac{2}{5}-\ln\frac{1}{5}$ $5\ln 2$ $\ln 32$ 3.466

EXERCISE 2 (12 PTS)

The 719 pupils in a high-school have been surveyed and asked the number of books they have read and films they have watched during the past year. Here is the result.

Nb of films	0	1	2	3	4	5	6	7	8	
Nb of students	5	10	10	20	35	40	60	80	85	
	9	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
	80	70	60	50	40	30	20	10	8	6

	Books		
	0 to 4	5 to 9	10 to 14
S	120	59	80
ES	50	100	50
L	50	110	100

Part A

1. Among the surveyed pupils, what is the percentage (rounded to the unit) of those reading less than 4 books?
2. Among those reading more than 10 books what is the percentage being in L ?
3. Among the S pupils, what is the percentage of those reading more than 4 books ?

Part B

1. Calculate the average $\overline{x}$ and the standard deviation σ of the data set of the number of films watched.
2. a. Calculate the interval $[\overline{x}-\sigma;\overline{x}+\sigma]$ and then $[\overline{x}-2\sigma;\overline{x}+2\sigma]$.
   b. For each of these intervals, calculate the percentage of the highschool's pupils whose number of films watched belongs to the interval.

Part C

We **now** assume that the random variable X equal to the number of films watched follows a Gaussian distribution and the previous results lead us to take $\mu = 9$. The parameter σ is yet unknown.

1. We know that the probability that a pupil has watched <u>at most</u> 8 films is 0.362.
   a. Translate the previous sentence in terms of probabilities.
   b. What is probability distribution of the variable $Z = \dfrac{X-9}{\sigma}$?
   c. Prove that $P(X \leq 8) = P\left(Z \leq -\dfrac{1}{\sigma}\right)$.
   d. The calculator gives $P(Z \leq -0.353) \approx 0.362$. Deduce the value of σ (rounded to the unit)
2. Calculate the probability that a random pupil has watched **at least** one film every month.
3. Calculate the probability that a random pupil has watched **at most** two films every month.
4. Calculate the probability that a random pupil has watched **between** 5 and 9 films in one year.