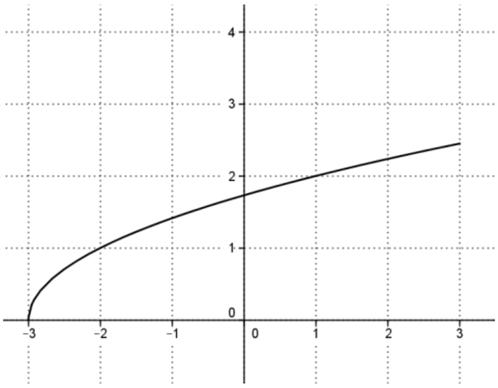
CONVEXITY AND INFLECTION EXERCISES

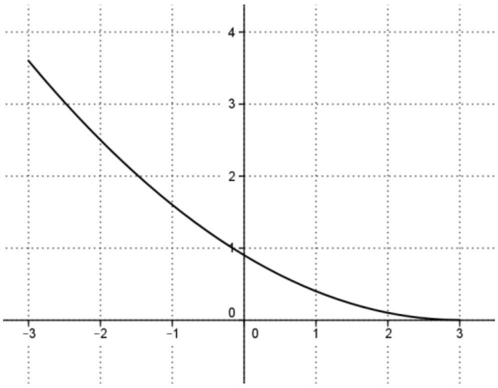
Exercice 1

For each curve, give the table of variation of f and f.

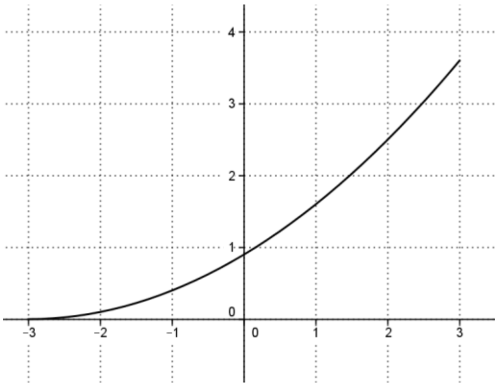
a.



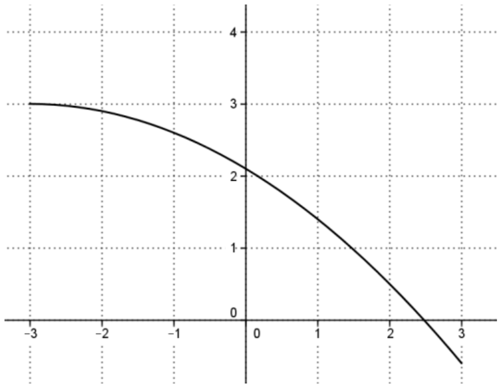
b.



c.



d.



Exercice 2

For each of the following quadratics, study their concavity :

a. $f(x)=0.5x^2+2x-1$ b. $g(x)=-0.5x^2-x+3.5$

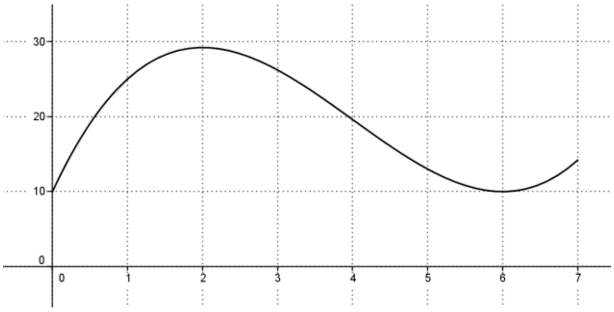
Exercice 3

Consider the function f defined on $[-3;3]$ by : $f(x)=x^3-3x+1$ and $\mathcal{C}$ its graph.

1. Work out $f'(x)$ and $f''(x)$.
2. Find the root x_0 of f''. Deduce the sign of $f''(x)$ on $[-3;3]$.
3. Does $\mathcal{C}$ have an inflection point ?

Exercice 4

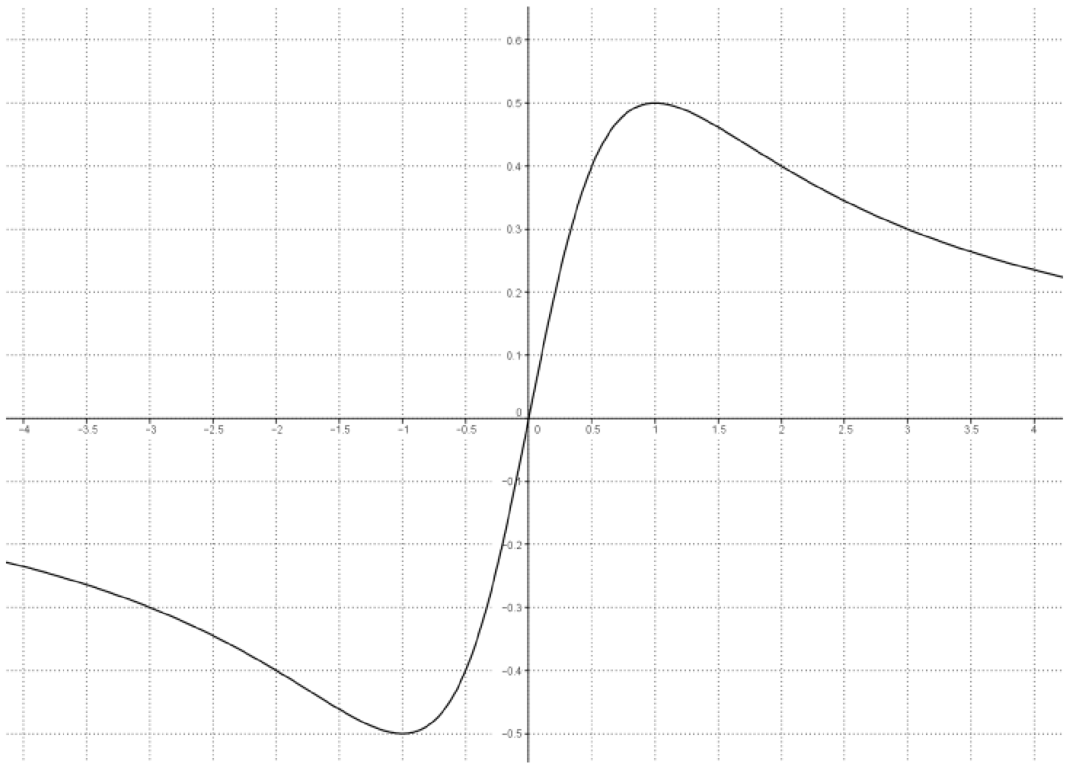
The value of one share of a tourism company can be modelized between 2005 and 2012 by the function f defined on $[0;7]$ by : $f(x)=0.6x^3-7.2x^2+21.6x+10$. Its graph $\mathcal{C}$ is drawn opposite. x represents the number of years since 2005.



1. a. Calculate the value of the share at the beginning of 2005.
   b. Study the variations of f.
   c. Work out which year the share will reach back its level of 2005.
2. Calculate $f''(x)$ and prove the graph has an inflection point. Give an interpretation of it.

Exercice 5

Consider the function f defined on $\mathbb{R}$ by : $f(x)=\dfrac{x}{x^2+1}$ and $\mathcal{C}$ its graph.



1. Graphically, study the concavity of f depending on the value of x.
2. Deduce the existence of three inflection points.
3. Calculate the derivative of f and check that $f'(x)=\dfrac{1-x^2}{\left(x^2+1\right)^2}$
4. Work out the equation of the tangent T_0 at 0.
5. We denote $d(x)=x-f(x)$. Check that $d(x)=\dfrac{x^3}{x^2+1}$ and deduce the sign table of $d(x)$.
6. Deduce the relative position of T_0 and $\mathcal{C}$.
7. Is O an inflection point ?

Exercice 6

Consider the function f defined in the previous exercise.

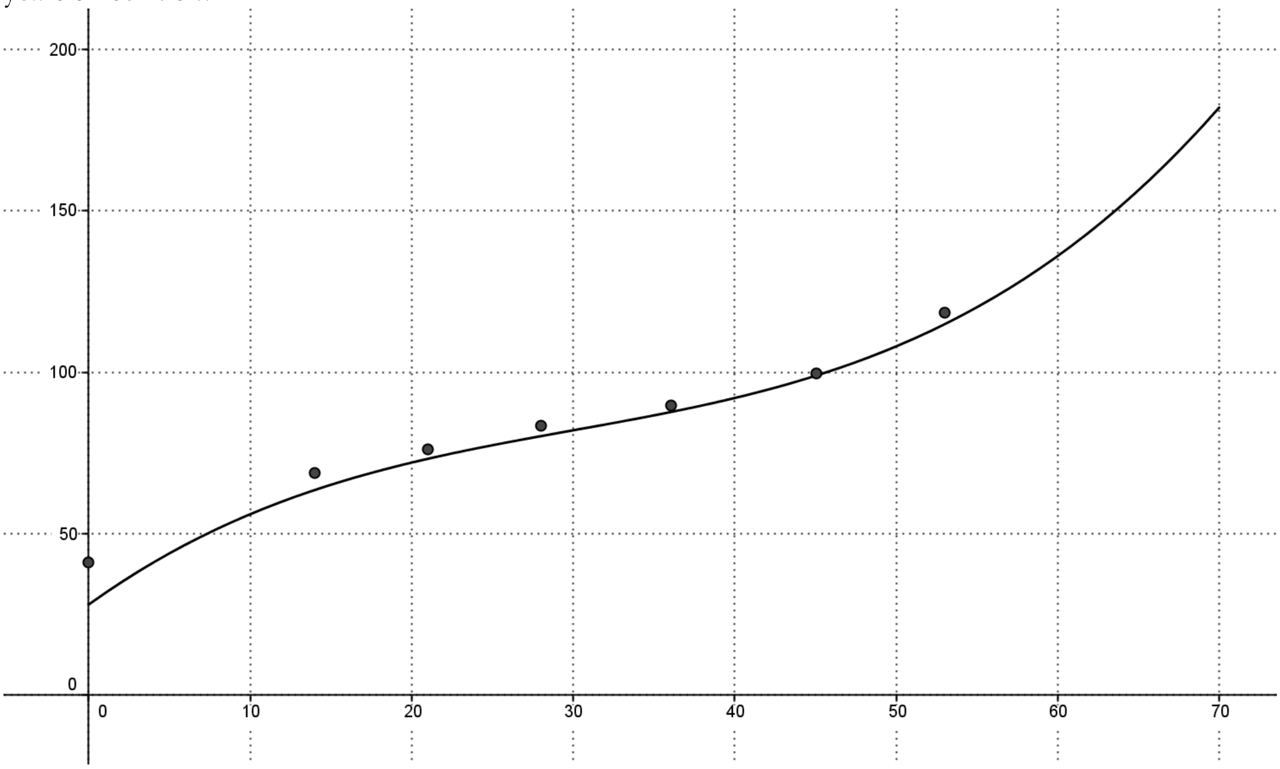
1. Using your calculator or a graphing software, give the table of variation of f'.
2. Deduce the concavity of f.
3. Prove that $f''(x)=\dfrac{2x\left(x^2-3\right)}{\left(x^2+1\right)^3}$.
4. Work out the sign of $f''(x)$ and make the link with the variations of f'.
5. Find the three inflection points of $\mathcal{C}$.

Exercice 7 **Pace of growth**

The table below give the urban area (in thousands of km²) in France, since 1954.

Years	1954	1968	1975	1982	1990	1999	2007
Urban area	41.1	68.9	76.3	83.4	89.6	100	118.8

The value of this area can be modelized between 1954 and 2020 by the function S defined on $[0;70]$ by : $S(x)=0.001x^3-0.09x^2+3.6x+28$. Its graph $\mathcal{C}$ is drawn opposite. x represents the number of years since 1954.



The **growing speed** of the urban area is considered to be equal to $S'(x)$.

1. a. Calculate the estimated value (according to this model) of the urban area in 1990.
   b. Give in % the relative error compared to tha actual value.
   c. Estimate the urban area in 2015.
2. a. Calculate $S'(x)$ and then $S''(x)$.
   b. Justify that S is increasing on $[0;70]$. Interpret this fact.
3. Which year is the growing speed the slowest ? What represents this year for the function S ? Check graphically this result.