CONTINUOUS PROBABILITY DISTRIBUTIONS EXERCISES SHEET

Ecole Internationale PACA à Manosque Tle ES

Exercise 1

Is it possible for the function f defined on $[1;10]$ by $f(x)=\dfrac{2}{x^3}$ to be a density function ?

Exercise 2

1. Find the real number k such that the function $f(x)=ke^{-x}$ is a density function on $[0;1]$.
2. Calculate then $P(X \geq 0.5)$.

Exercise 3

Two friends have a date in a restaurant between 12am and 13 pm. Jean decides to arrive at 12h30 and Jude arrives randomly between 12h and 13h.

a. What is the probability distribution of the random variable equal to the arrival time of Jude ?
b. Calculate the probability Jude arrives before Jean.
c. Calculate the probability for Jean to wait more than 15'.

Exercise 4

Between 5h and 24h, one TGV leaves the "Gare de Lyon" station every 2 hours. Ziva has to take one of these trains. She arrives at the station between 8h and 10h. We denote X the random variable equal to this arriving time of Ziva.

1. Give the interval on which X follows a uniform distribution.
2. Calculate the probability for Ziva to wait for her train less than 30'.

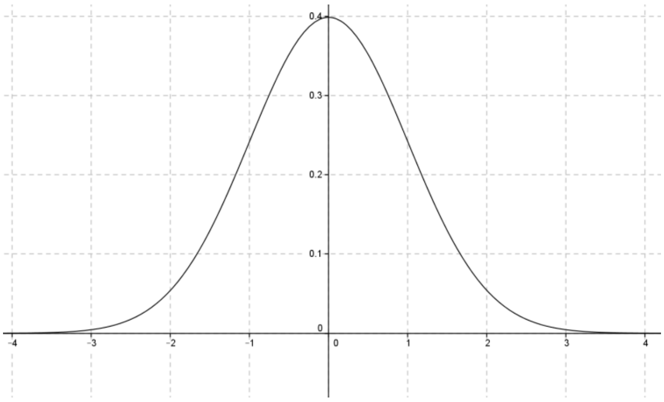
Exercise 5

A random variable follows a normal distribution $\mathcal{N}(0;1)$.

1. Calculate with your calculator $P(-0.5 \leq X \leq 0.5)$.
2. Deduce $P(X \geq 0.5)$.

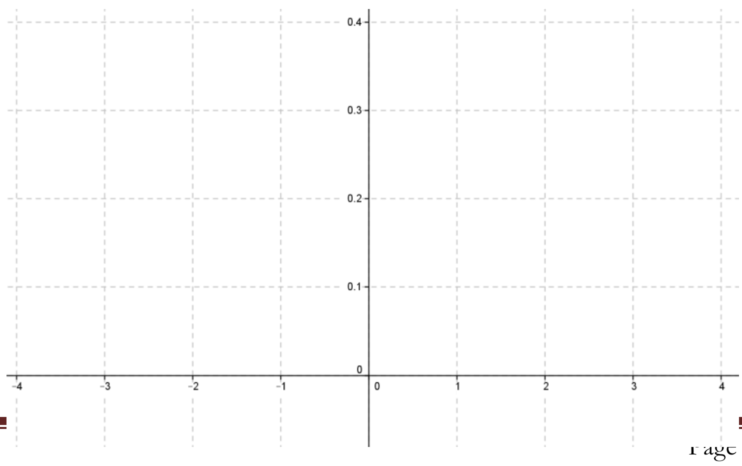
Exercise 6 TRUE/FALSE

If a random variable follows a normal distribution $\mathcal{N}(0;1)$ then :



1. Its density function is $f(x)=e^{-\frac{x^2}{2}}$
2. The graph of the density function is symmetrical about the y-axis
3. The density function is defined only on [-3;3]
4. The values for $x \leq -3$ and $x \geq 3$ do not appear since they are smaller than 0.01 .

Exercise 7



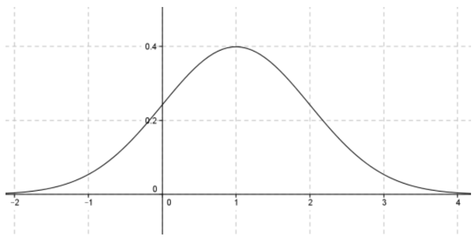
1. Thanks to your calculator draw up opposite the graph of the density function of $\mathcal{N}(0;1)$.
2. Make appear $P(0 \leq X \leq 1)$ on your graph . Give an approximate value of this probability.
3. Deduce $P(-1 \leq X \leq 1)$.

Exercise 8

Associate each graph with its probability distribution.

a) $\mathcal{N}(-2;1)$ b) $\mathcal{N}(1;4)$ c) $\mathcal{N}(1;1)$ d) $\mathcal{N}(5;4)$

1.



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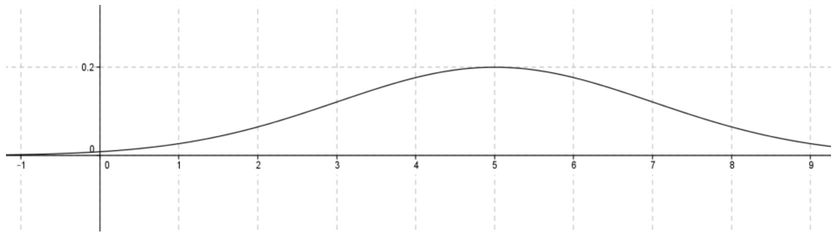

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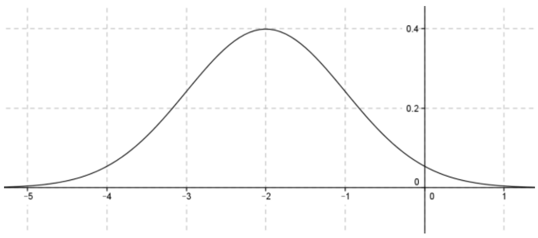

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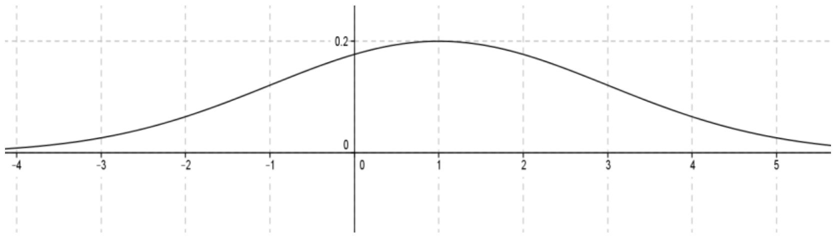

Exercise 9

A machine is filling juice bottles. Because of variations in the various mechanisms, the weight of the bottle varies and follows a normal distribution with parameters $\mu = 1000$ g and $\sigma = 5$.

Calculate the probability for a bottle to weigh between 990 and 1010 g.

Exercise 10

A survey has showed that, yesterday evening, 78% of the pupils have watched Doctor H.

We interview successively and independently 25 pupils. Let X represents the random variable associated with the number of pupils having watched Doctor H yesterday.

1. True or False (Justify)

   a. $X \sim \mathcal{B}(0.78;25)$ b. $P(X=18) = \binom{25}{18} \times 0.78^{18} \times 0.22^{7}$ c. $P(X=25) = 0.78^{25}$

2. Thanks to the calculator, give :

   a. $P(X=18)$ b. $P(X \leq 20)$ c. $P(X > 20)$ d. $P(18 \leq X \leq 20)$

3. True or False (Justify)

   a. $E(X) = \frac{25}{2}$ b. $E(X) = 25 \times 0.78$ c. $E(X) = 25 + 0.78$ d. $V(X) = 4.29$

Exercise 11

The life expectancy X (in hours) of an electric bulb follows $\mathcal{N}(\mu;\sigma^2)$. We know that $P(X \geq 10000) = 0.6$ and $P(X \leq 13000) = 0.69$.

1. What is probability distribution of the variable $Z = \frac{X-\mu}{\sigma}$?
2. Prove that $P\left(Z \geq \frac{10000-\mu}{\sigma}\right) = 0.6$ and $P\left(Z \leq \frac{13000-\mu}{\sigma}\right) = 0.69$
3. Justify that μ and σ are solutions of the simultaneous equations $\begin{cases} 10000-\mu = -0.252 \times \sigma \\ 13000-\mu = 0.496 \times \sigma \end{cases}$
4. Deduce the values of μ and σ (rounded to the unit).

Exercise 12

On average the width of an adult man's hand is 9.5 cm. It follows $\mathcal{N}(9.5;4)$. Results will be rounded to 1 mm. A door factory studies this distribution to set up his production.

1. What is the probability that one random man has a hand less than 8cm wide ?
2. What is the probability that one random man has a hand more than 12cm wide ?
3. The factory wants his production to fit at least 90% of the population.
   a. Which interval with amplitude 2a centered at 9.5 has a probability equal to 0.9?
   b. What must be the minimal length of the door handle ?

Exercise 13 Height of women in France

1. A study, conducted on a sample containing as many women as men, has showed that 7% of women and 87% of men measure more than 1.75 m. We choose randomly a person in this sample.
   a. Calculate the probability that this person is a woman measuring more than 1.75m.
   b. Calculate the probability that this person measure more than 1.75m.
   c. We choose one person measuring more than 1.75m. What is the probability that it's a woman ?
2. We now assume that the random variable X equal to the height of French woman follows a Gaussian distribution with $\mu = 163$ cm and $\sigma = 11$. See table opposite.
   a. Calculate the probability that a French woman measure less than 1.75m.
   b. The minimum height for some jobs is 1.75cm. What is the probability a French woman is eligible for this kind of job ?
   c. Find the biggest number a such that : $P(X \le a) = 0.70$.
   d. Deduce the maximal height (to the cm) of 70% of French women.

a	$P(X \le a)$
164	0,5362
165	0,5721
166	0,6075
167	0,6419
168	0,6753
169	0,7073
170	0,7377
171	0,7665
172	0,7934
173	0,8183
174	0,8413
175	0,8623
176	0,8814
177	0,8984
178	0,9137
179	0,9271
180	0,9389
181	0,9491
182	0,9579
183	0,9655
184	0,9719
185	0,9772
186	0,9817
187	0,9854
188	0,9885

Exercise 14

The 719 pupils in a high-school have been surveyed and asked the number of books they have read and films they have watched during the past year. Here is the result.

Nb of films	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
Nb of students	5	10	10	20	35	40	60	80	85	
	9	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
	80	70	60	50	40	30	20	10	8	6

	Books		
	0 to 4	5 to 9	10 to 14
S	120	59	80
ES	50	100	50
L	50	110	100

Part A

1. Among the surveyed pupils, what is the percentage (rounded to the unit) of those reading less than 4 books?
2. Among those reading more than 10 books what is the percentage being in L ?
3. Among the S pupils, what is the percentage of those reading more than 4 books ?

Part B

1. Calculate the average $\overline{x}$ and the standard deviation σ of the data set of the number of films watched.
2. a. Calculate the interval $\left[\overline{x}-\sigma;\overline{x}+\sigma\right]$ and then $\left[\overline{x}-2\sigma;\overline{x}+2\sigma\right]$.
   b. For each of these intervals, calculate the percentage of the highschool's pupils whose number of films watched belongs to the interval.

Part C

We now assume that the random variable X equal to the number of films watched follows a Gaussian distribution and the previous results lead us to take $\mu=9$. The parameter σ is yet unknown.

1. We know that the probability that a pupil has watched at most 8 films is 0.362.
   a. Translate the previous sentence in terms of probabilities.
   b. What is probability distribution of the variable $Z=\frac{X-9}{\sigma}$?
   c. Prove that $P\left(X\leq 8\right)=P\left(Z\leq -\frac{1}{\sigma}\right)$.
   d. The calculator gives $P\left(Z\leq -0.353\right)\approx 0.362$. Deduce the value of σ (rounded to the unit)
2. Calculate the probability that a random pupil has watched at least one film every month.