EXPONENTIAL FUNCTIONS
EXERCISES

Ecole Internationale PACA à Manosque Tle ES

Exercise 1

Find which numbers are strictly positive.

a) $\exp(-2)$ b) 3.1^{-1} c) -1.05^2 d) $\exp\left(\frac{1}{3}\right)$ e) $\exp(-100)$ f) $3-e$

Exercise 2

1. Simplify. a) $e^3 \times e^{x-2}$ b) $2^{-2x} \times 2^{x+3}$ c) $\frac{1-e^{2x}}{e^x}$

d) $(3^{0.6})^3 \times 3^{-0.8}$ e) $\frac{1.4^{x+0.3} \times 1.4^{x+0.7}}{1.4^{2x}}$ f) $\frac{\exp(1+x)}{\exp(1-x)}$

2. Expand. $f(x) = (3^x+3)(1-3^x)$ $g(x) = (1+e^x)(1-e^x)$

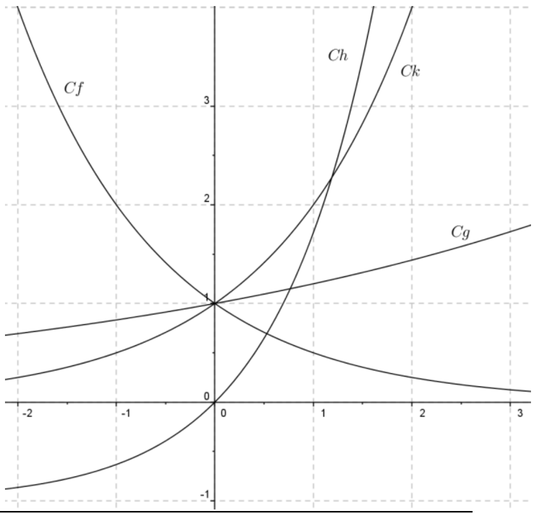
Exercise 3

Let's consider the function g defined on $\mathbb{R}$ by $g(x) = 1-2.5^x$.

1. Calculate $g(-2)$, $g(-1)$, $g(0)$, $g(1)$, $g(2)$
2. Speculate the sign of $g(x) = 1-2.5^x$ with regards to x.
3. Using your calculator and the graph of the function $f(x) = 2.5^x$, prove the previous hypothesis.

Exercise 4

1. Among the graphs opposite, one is not the one of an exponential function, which one ?
2. Find the value of q for the three remaining functions.



Exercise 5

Let's consider the function f defined on $\mathbb{R}$ by $f(x) = x\frac{1-e^x}{1+e^x}$

1. Compare $f(1)$ and $f(-1)$.
2. Prove that $f(-x) = f(x)$ for any real number x.

Exercise 6

Solve the following equations/inequations.

a) $2e^x(e^x-1) = 0$ b) $(e^x - e)(e^x+2) = 0$ c) $\frac{1-e^x}{1+x^2} = 0$ d) $\frac{5+e^x}{1+e^x} = 3$

e) $e^{2x} - e^x = 0$ f) $e^{1+x} = e^{-x}$ g) $e^{2x} - 4e^x < 0$ j) $e^{-x+1} < e^{2x-5}$ l) $e^{0,5x+1} \geq 1$

Exercise 7

Find the derivative of the following functions.

a) $f(x) = xe^x$ b) $f(x) = (3x^2-2x+1)e^x$ c) $f(x) = \frac{e^x-2}{e^x+1}$ d) $f(x) = e^{2-x}$ e) $f(x) = x-2e^x$

f) $f(x) = \frac{3}{2}e^{2x} - e^x - 2x + 4$ g) $f(x) = \frac{8(e^x-1)}{e^{2x}}$ h) $f(x) = \frac{1+e^x}{1-e^x}$ i) $f(x) = 0.5e^{x^2-5x+1}$

Exercise 8

Let f be the function defined on ℝ by : $f(x)=(x^2-3)e^x$

1. Study it's variations
2. Solve $f(x)=0$
3. Deduce the sign of $f(x)$ with regards to x.

Exercise 9

Study the following functions on ℝ.

a) $f(x)=e^x-x+1$ b) $f(x)=(x+2)e^{-x}$ c) $f(x)=xe^x-ex$ d) $f(x)=\dfrac{e^x}{2+e^{-x}}$

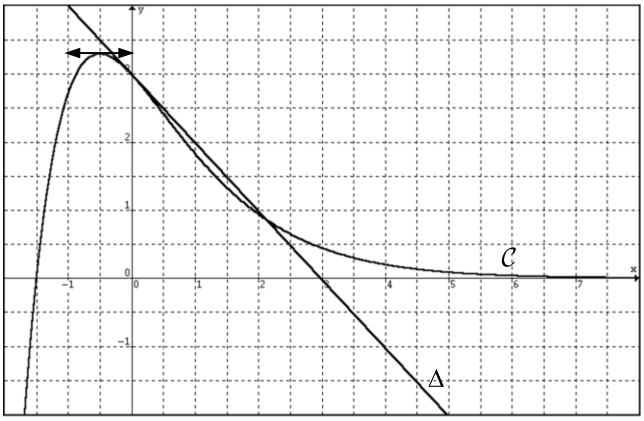
Exercise 10

The graph represents the curve of the function f defined on ℝ by:

$f(x)=e^{-x}(ax+b)$ with a and b real numbers.

The line Δ is a tangent of $\mathcal{C}$ at 0.

The tangent at the point $A\left(-\frac{1}{2};\frac{7}{2}\right)$ is drawn as well.



Part A :

1)Using the graph, find :

$f\left(-\frac{3}{2}\right)$, $f(0)$, $f'\left(-\frac{1}{2}\right)$. Calculate $f'(0)$.

2)Find the equation of the line Δ.

3)Find the values of a and b.

Part B :

1)Let h be a function defined on ℝ by : $h(x)=-e^{-x}(2x+1)+1$. Draw up the table of variations of h. Then, deduce the sign of h(x) on [0 ; 1].

2)Let g be a function defined on ℝ by : $g(x)=e^{-x}(2x+3)+x-3$. Calculate $g'(x)$ thanks to h(x). Deduce the variations of g on [0 ; 1] and then, deduce the sign of g(x) on [0 ; 1].

3)Prove that $\mathcal{C}$ is above Δ on [0 ; 1].

Exercise 11

Let's consider the functions f and g defined on ℝ by $f(x)=x(1+e^{-x})$ and $g(x)=1+(1-x)e^{-x}$.

1. The table of variations of g is as follows :

x	$-\infty$		2		$+\infty$
g		↘	$1-e^{-2}$	↗	

What is the sign of g(x) ?

2. a. Show that, for any x, $f'(x)=g(x)$
   b. Deduce the sense of variation of f
3. Consider the line (d) with equation $y=x$. Study the relative position of (d) with regards to C_f.
4. a. Solve the equation $f'(x)=1$
   b. deduce that C_f has a unique tangent T which is parallel to (d)
   c. work out an equation of T
5. We denote $h(x)=f(x)-(x+e^{-1})$
   a. Study the variations of h.
   b. Deduce it's sign

Exercise 11 correction

Let's consider the functions f and g defined on $\mathbb{R}$ by $f(x)=x\left(1+e^{-x}\right)$ and $g(x)=1+(1-x)e^{-x}$.

1. $1-e^{-2}\approx 0.86>0$, thus g is always positive.

2. a. For any x, $f'(x)=1\times\left(1+e^{-x}\right)+x\left(-e^{-x}\right)=1+e^{-x}-xe^{-x}=1+e^{-x}(1-x)=g(x)$

   b. f' having the same sign as g, is always positive, hence f is always increasing on $\mathbb{R}$.

3. To study the relative position of (d) with regards to C_f, lets study the sign of $f(x)-x$.
   $f(x)-x=x\left(1+e^{-x}\right)-x=xe^{-x}$ which has the same sign as x (since $e^{-x}>0$), that's positive right from 0 and negative left. C_f is thus above (d) for $x>0$ and below for $x<0$.

4. a. $f'(x)=g(x)=1$ when $(1-x)e^{-x}=0$ thus when x=1.

   b. The point A with x-coordinate 1 is thus the only one for which the gradient of the tangent is 1 and thus where the tangent is parallel to (d).

   c. work out an equation of T

5. $h(x)=f(x)-\left(x+e^{-1}\right)=f(x)=x\left(1+e^{-x}\right)-\left(x+e^{-1}\right)=xe^{-x}-e^{-1}$

   a. $h'(x)=....=e^{-x}(1-x)$ and has thus the sign of 1-x, hence the table of variation :

x	$-\infty$		1		$+\infty$
$1-x$		+	0	−	
h		↗	0	↘	

   b. $h(x)$ is thus always negative