ln FUNCTION
Exercises sheet

Exercise 1 : True or false

a) for any real number $a > 0$, $ln(3a) - ln(a) = ln 3$

b) $\dfrac{ln(e^2)}{ln 16} = 2 - ln 16$

c) $ln(e^2 + e) = 1 + ln(e+1)$

d) for any real number x , $ln(x^2 + x) = ln\, x + ln(x+1)$

Exercise 2 :

Solve the equations :

a) $e^x = 2$ b) $e^x + 2 = 3$ c) $e^{-0.03x} = 0.25$ d) $(e^x - 1)(3e^x - 4) = 0$

e) $ln\, x + 1 = 0$ f) $(x+2) ln\, x = 0$ g) $ln\, x = 2 ln 3$ h) $ln(x^2 + x + 1) = 0$

Exercise 3 :

Express with a single logarithm :

a) $2 ln 10 - 3 ln 5 + ln 2$ b) $7 ln 4 - 3 ln 2 - 4 ln 8$ c) $ln \dfrac{1}{2} - 4 ln 2 + ln 16$

Exercise 4 :

Let's consider the function defined by $f(x) = e^{2x} - 5e^x + 4$:

1. Work out $f'(x)$.
2. Solve in $\mathbb{R}$ the inequation $2e^x - 5 \geq 0$.
3. Deduce the table of sign of $f'(x)$ and the variations of f.

Exercise 5 :

In France, since 1970, the consumption of sugar per inhabitant (in kg/year) can be modelized by the function $f(x) = 20e^{-0.03x}$ where x is the rank of the year $1970 + x$.

1. Calculate $f(50)$.
2. Justify the sense of variation of this function.
3. a. Solve $f(x) = 5$ and then $f(x) \leq 5$.

   b. Deduce the year where the sugar consumption will be less than 5 kg/inh/year.
4. Draw up the graph of f (units : 1cm for 10 years and 1 cm par 5 kg/inh/year).

Exercise 6 : True/False

1. If $f(x) = 3x\, ln\, x$, then $f'(x) = 3$.
2. If $f(x) = x(1 - ln\, x)$, then $f'(x) = -ln\, x$.
3. If $f(x) = \dfrac{3 ln\, x}{x}$, then $f'(x) = \dfrac{3(1 - ln\, x)}{x^2}$.
4. If $f(x) = (ln\, x)^2 - ln\, x$, then $f'(x) = \dfrac{2 ln\, x - 1}{x}$.

Exercise 7 :

Solve the inequations :

a) $ln\,x+2\geq 0$ b) $2\,ln\,x+1>0$ c) $(2-ln\,x)ln\,x<0$

Exercise 8 :

Calculate the derivatives of the following functions (check your result with a CAS software):

a) $f(x)=ln\,x+x$ b) $f(t)=(ln\,t+1)(ln\,t-2)$ c) $f(x)=x^2\,ln\,x$

d) $f(x)=\dfrac{ln\,x+1}{x}$ e) $f(t)=0.5(ln\,t)^2-t$ f) $f(x)=x\,ln\,x+3$

Exercise 9 :

Study the variations of the following functions on the given interval :

a) $f(x)=2x-ln\,x$ on $[1;9]$ b) $f(x)=x\,ln\,x-x$ on $[0.01;10]$

Exercise 10 :

1. Solve the equations and inequations:

a) $x^{10}=2$ b) $x^3=0.5$ c) $(1+x)^4=1.5$ d) $3\times 1.5^x\geq 12000$

2. Find the smallest integer n such that :

a) $0.8^n\leq 0.001$ b) $10\times 1.1^n\geq 5000$ c) $1-0.4^n\geq 0.999$

Exercise 11 :

1. Solve the equation : $1.1^n=2$
2. Deduce the number of years required for an investment placed at a 10% yearly rate to double.
3. Solve the inequation : $0.85^n<0.5$
4. Deduce the number of years required for population decreasing by a 15% yearly rate to reduce by 50%.
5. Check with the algorithm made in the lesson on sequences (threshold exceed)

Exercise 12 :

We roll n times a fair die (the different throws are considered as independent) and denote $P(n)$ the probability of getting 4, for the first time, at the nth roll.

1. What kind of sequence is $(P(n))$?
2. What is it's limit ?
3. Find the smallest integer n such that $P(n)<0.0001$
4. Check with your calculator

Exercise 13 : Mean increase rate

1. The mineral water consumption in France has risen from 133.7 L/inh in 1998 to 151.1 L/inh in 2008.
   a. Calculate the corresponding multiplier with 4 dp.
   b. Justify that the yearly average rate t satisfy the equation : $(1+t)^{10}=\dfrac{151.1}{133.7}$
   c. Work out t.
   d. If this rate remains steady, give an estimation of the consumption in 2015.
2. Same questions for the medical care consumption with has risen from 64.7 billion € in 1998 to 176 billion € in 2008