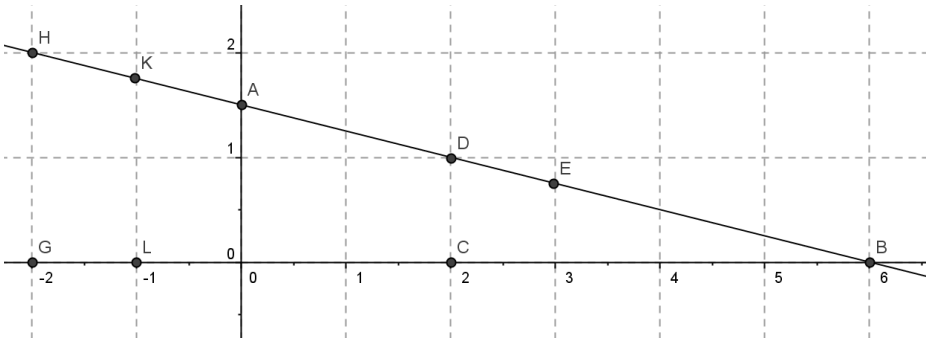
TP
INTRODUCTION TO INTEGRATION

Ecole Internationale PACA à Manosque Tle S

ONE SPECIAL CASE

Let f be the function defined by $f(x) = -0.25x + 1.5$.

Its graph is given in a coordinate system with units' length 1 cm.



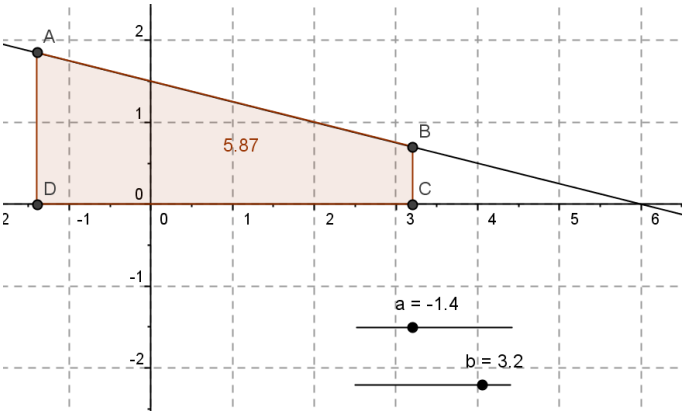
BY HAND

1. Calculate the area of the quadrilaterals OAB, CDEM, GHKL and CDKL.
2. Find a primitive (antiderivative) F of f .
3. Calculate $F(6) - F(0)$ and compare the result to the area of OAB.
4. Calculate $F(3) - F(2)$ and compare the result to the area of CDEM.
5. How could you find the areas of GHKL and CDKL using F ?

USING GEOGEBRA

Make the opposite figure, a being the x-coordinate of A and b the one of B, so you can make vary A and B on the line using the sliders a and b.

1. Define F(x) (F= Intégrale[f]).
2. Display (in the algebra window) $F(b) - F(a)$ and check it remains equal to the area of ABCD.

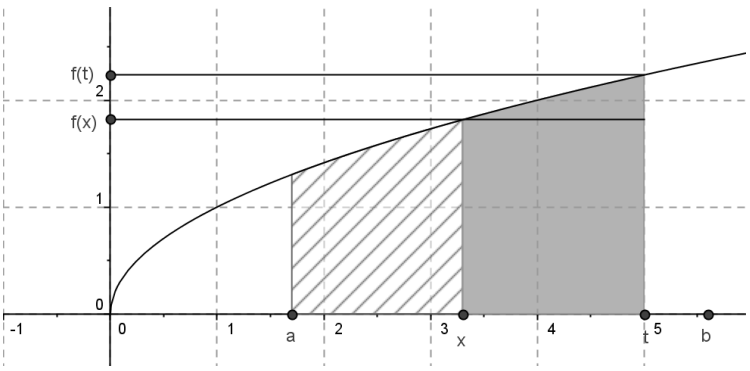


JUSTIFICATION

Let's consider a function f, continuous, increasing and positive on an interval $I = [a;b]$ and its graph $\mathcal{C}$.

For any x in I, let's denote F(x) the area below $\mathcal{C}$ between a and x (grey stripes).

Let's denote t a number such that $x < t \leq b$



1. Explain why the grey area is $F(t) - F(x)$?
2. Explain why $f(x)(t-x) \leq F(t) - F(x) \leq f(t)(t-x)$?
3. Deduce then that $f(x) \leq \dfrac{F(t) - F(x)}{(t-x)} \leq f(t)$
4. What is $\lim\limits_{t \to x} f(t)$?
5. Deduce then why F is differentiable on I and that $F'(x) = f(x)$.
6. What is then this function F for f ?