INSTRUCTIONS TO CANDIDATES

• Write your session number in the boxes above.
• Do not open this examination paper until instructed to do so.
• A graphic display calculator is required for this paper.
• Section A: answer all of Section A in the spaces provided.
• Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
• At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
• Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer all the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Let \( A = \begin{pmatrix} 1 & 2 & -3 \\ -1 & -1 & 4 \\ 2 & 4 & -3 \end{pmatrix} \) and \( B = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \).

(a) Write down \( A^{-1} \). [2 marks]

(b) Solve \( AX = B \). [3 marks]
2. [Maximum mark: 6]

Consider the arithmetic sequence 3, 9, 15, ..., 1353.

(a) Write down the common difference. [1 mark]

(b) Find the number of terms in the sequence. [3 marks]

(c) Find the sum of the sequence. [2 marks]
3. \([\text{Maximum mark: 7}]\)

Let \( f(x) = x \cos x \), for \( 0 \leq x \leq 6 \).

(a) Find \( f'(x) \). \([3 \text{ marks}]\)

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(b) On the grid below, sketch the graph of \( y = f'(x) \). \([4 \text{ marks}]\)

\[\begin{array}{|c|c|c|c|c|c|c|}
\hline
x & -1 & 0 & 1 & 2 & 3 & 4 \\
\hline
y & & & & & & \\
\hline
\end{array}\]
4. [Maximum mark: 6]

The following frequency distribution of marks has mean 4.5.

<table>
<thead>
<tr>
<th>Mark</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>9</td>
<td>x</td>
<td>9</td>
<td>4</td>
</tr>
</tbody>
</table>

(a) Find the value of $x$. [4 marks]

(b) Write down the standard deviation. [2 marks]
5. [Maximum mark: 7]

The graph of \( y = p \cos qx + r \), for \(-5 \leq x \leq 14\), is shown below.

There is a minimum point at \((0, -3)\) and a maximum point at \((4, 7)\).

(a) Find the value of

(i) \( p \);

(ii) \( q \);

(iii) \( r \). [6 marks]

(b) The equation \( y = k \) has exactly two solutions. Write down the value of \( k \). [1 mark]
6. [Maximum mark: 7]

The acceleration, \( a \text{ m s}^{-2} \), of a particle at time \( t \) seconds is given by

\[
a = \frac{1}{t} + 3\sin 2t, \text{ for } t \geq 1.
\]

The particle is at rest when \( t = 1 \).

Find the velocity of the particle when \( t = 5 \). 

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7. [Maximum mark: 7]

Evan likes to play two games of chance, A and B.

For game A, the probability that Evan wins is 0.9. He plays game A seven times.

(a) Find the probability that he wins exactly four games. [2 marks]

For game B, the probability that Evan wins is \( p \). He plays game B seven times.

(b) Write down an expression, in terms of \( p \), for the probability that he wins exactly four games. [2 marks]

(c) Hence, find the values of \( p \) such that the probability that he wins exactly four games is 0.15. [3 marks]
Do NOT write on this page.

SECTION B

Answer all the questions on the answer sheets provided. Please start each question on a new page.

8. [Maximum mark: 14]

The diagram below shows a quadrilateral ABCD with obtuse angles $\hat{A}BC$ and $\hat{A}DC$.

![Diagram of quadrilateral ABCD](attachment:diagram.png)

AB = 5 cm, BC = 4 cm, CD = 4 cm, AD = 4 cm, $\angle BAC = 30^\circ$, $\hat{A}BC = x^\circ$, $\hat{A}DC = y^\circ$.

(a) Use the cosine rule to show that $AC = \sqrt{41 - 40\cos x}$.

(b) Use the sine rule in triangle ABC to find another expression for AC.

(c) (i) Hence, find $x$, giving your answer to two decimal places.

(ii) Find AC.

(d) (i) Find $y$.

(ii) Hence, or otherwise, find the area of triangle ACD.
Let \( f(x) = Ae^{kx} + 3 \). Part of the graph of \( f \) is shown below.

The \( y \)-intercept is at \((0, 13)\).

(a) Show that \( A = 10 \).  

(b) Given that \( f(15) = 3.49 \) (correct to 3 significant figures), find the value of \( k \).  

(c) (i) Using your value of \( k \), find \( f'(x) \).

(ii) Hence, explain why \( f \) is a decreasing function.

(iii) Write down the equation of the horizontal asymptote of the graph \( f \).  

Let \( g(x) = -x^2 + 12x - 24 \).

(d) Find the area enclosed by the graphs of \( f \) and \( g \).
10.  [Maximum mark: 15]

The weights of players in a sports league are normally distributed with a mean of 76.6 kg, (correct to three significant figures). It is known that 80% of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

(a) Find the probability that a player weighs more than 82 kg.  

(b) (i) Write down the standardized value, \(z\), for 68 kg.

(ii) Hence, find the standard deviation of weights.

To take part in a tournament, a player’s weight must be within 1.5 standard deviations of the mean.

(c) (i) Find the set of all possible weights of players that take part in the tournament.

(ii) A player is selected at random. Find the probability that the player takes part in the tournament.

Of the players in the league, 25% are women. Of the women, 70% take part in the tournament.

(d) Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman.