



PROBA TP N°5: Algorithms and decision making from sampling

Objective: Find the interval corresponding to the acceptance region using an algorithm

S is a random variable which follows a binomial distribution with parameters n and p .

1. Finding a

Given the algorithm opposite:

a) Work through the algorithm “by hand” for $n = 10$ and $p = 0.7$ and thus progressively complete the table below for successive values of k and S .

| k | $P(X = k)$ |
|-----|-------------|
| 0 | 5.9049E-06 |
| 1 | 0.000143686 |
| 2 | 0.001590386 |
| 3 | 0.010592078 |
| 4 | 0.047348987 |
| 5 | 0.150268333 |
| 6 | 0.350389282 |
| 7 | 0.617217214 |
| 8 | 0.850691654 |
| 9 | 0.971752475 |
| 10 | 1 |

To help you, you could use the following data:

Input

input n and p
 e takes the value 0.05
 k takes the value 0
 S takes the value 0

Process

While $S \leq \frac{e}{2}$
 S takes the value $S + P(X = k)$
 k takes the value $k + 1$

End
 a takes the value $k - 1$

Output

Display a

| | | | | | |
|-----|---|-------------------------|--|--|--|
| S | 0 | 5.9049×10^{-6} | | | |
| k | 0 | 1 | | | |

What is the value of a displayed in this case?

- b) Explain the role of this algorithm. Give the significance of each of the variables.
- c) Code this algorithm in a programming language on the computer or your calculator.
- d) Test the program obtained with different values of n and p .

2. Finding b

In the same way, write the algorithm which will find the smallest whole number b such that $P(X \leq b) \geq 0.975$
 Code your algorithm in a programming language and test it.

3. Varying the significance level

- a) Modify the preceding programs so that the significance level is fixed at 15%.
- b) Test the changes that this brings.

Reproduce the spreadsheet below, which you can use to answer questions.

You want it to work for any values of n and p (so take their value from B3 and D3).

You need to enter formulae in B6, C6 and D6 and then copy down to line 1006. You will also need to change the format of the numbers in column B (giving them 9dps).

Reminder of some useful commands:

=SI(Test; Valeur si vrai; Valeur si faux)

- used in B6 to show the probabilities $P(X \leq k)$ when X has a binomial distribution with parameters n and p. Also used in C6 and D6

=LOI.BINOMIALE(X ; tentatives ; probabilité ; C)

ie (k ; n ; p ; 1) with Open Office Calc or (k ; n ; p ; VRAI) with Excel

- used in B6

| | A | B | C | D | E | F | G | H | I | |
|----|--------------------------------------------------------------------------------------------------------------------------------|-------------|---------------------------------------|-------------|---|-----------|------|---|---|--|
| 1 | Interval corresponding to the acceptance region at significance level 5% of a relative frequency under a binomial distribution | | | | | | | | | |
| 2 | Size of sample | | Supposed proportion in the population | | | | | | | |
| 3 | | n=40 | | p=0,16 | | | | | | |
| 4 | | | | | | | | | | |
| 5 | k | P(X<=k) | Finding a/n | Finding b/n | | | | | | |
| 6 | 0 | 0,000935775 | | | | | | | | |
| 7 | 1 | 0,008065489 | | | | | | | | |
| 8 | 2 | 0,034547286 | 0,05 | | | | | | | |
| 9 | 3 | 0,098439873 | 0,08 | | | Interval: | | | | |
| 10 | 4 | 0,211012527 | 0,1 | | | 0,05 | 0,28 | | | |
| 11 | 5 | 0,365397882 | 0,13 | | | | | | | |
| 12 | 6 | 0,536937165 | 0,15 | | | | | | | |
| 13 | 7 | 0,695640174 | 0,18 | | | | | | | |
| 14 | 8 | 0,820335396 | 0,2 | | | | | | | |
| 15 | 9 | 0,904785071 | 0,23 | | | | | | | |
| 16 | 10 | 0,954650593 | 0,25 | | | | | | | |
| 17 | 11 | 0,980554760 | 0,28 | 0,28 | | | | | | |
| 18 | 12 | 0,992478900 | 0,3 | 0,3 | | | | | | |