BREVET BLANC N°2

DU 11 MAI 2012

Session 2012

MATHEMATIQUES

Durée de l'épreuve : 2 heures

Les calculatrices électroniques de poche sont autorisées.

Numerical Activities

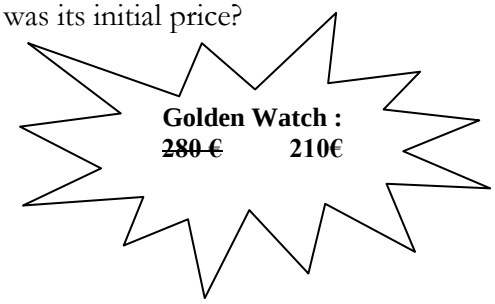
Exercice 1. ***(3 pts)***

John and Judy want to buy their mother a present for mother's day. They have 180€ altogether and want to take advantage of the sales period.

1. In a jewellery showroom, a beautiful pair of earrings is displayed at the price of 120€. What would be the price after a discount of 25% ?

2. There is also a ring whose price is 78.4€ after a discount of 20%. What was its initial price?

3. Just before leaving the jewellery, Judy sees a poster showing :
   What is the percentage discount on the golden watch?



Exercice 2. ***(4 pts)***

Consider : $D = 9x^2 - 4 + (3x-2)(x-3)$.

1. Expand and simplfy D.
2. Factorise $9x^2 - 4$ and then deduce the factorisation of D.
3. Calculate D for $x = -5$.
4. Solve the equation $(3x-2)(4x-1) = 0$.

Exercice 3. ***(3 pts)***

1. Write in the form $a\sqrt{7}$ (where a is an integer) the number B such that : $B = 4\sqrt{7} - 8\sqrt{28} + \sqrt{700}$.

2. Expand and simplify the expression C such that : $C = (4\sqrt{5} + 2)^2$.

GEOMETRICAL ACTIVITIES

Exercice 1. ***(6 pts)***

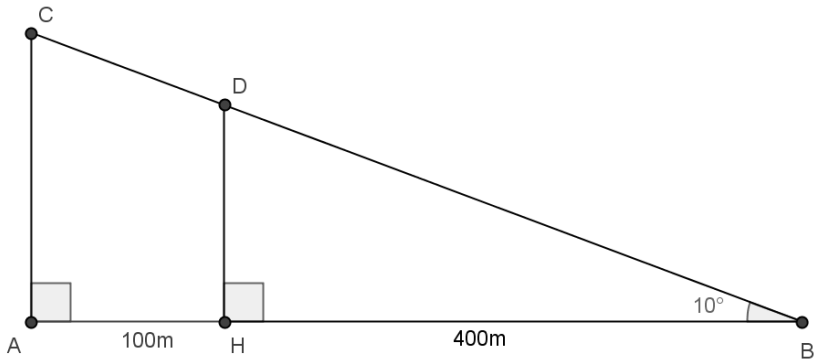
1. Draw a circle $\mathcal{C}$ with diameter [BC] such that BC = 10 cm. Mark on $\mathcal{C}$ a point A such that AB = 8 cm.
2. What kind of triangle is ABC ? Justify.
3. Calculate AC.
4. Find an approximate value (to one degree) of the angle $\widehat{ABC}$.
5. Mark the point O on segment [BC] such that BO = 2 cm. Draw the line parallel to the line (AC) that passes through B and cuts the line (AO) at D.
   Calculate BD.

Exercice 2. ***(6 pts)***

Every answer has to be justified.
A racing cyclist follows a track [CB].

$ABC = 10°$

The figure is not to scale ; you don't have to reproduce it.



1. Calculate BCA.
2. Calculate the difference in altitude AC rounded to the nearest meter
3. Calculate BC rounded to the nearest meter.
4. The racing cyclist has stopped at D on his way. Calculate the distance DB he has left to do (to the nearest meter).

Problems

Exercice 1. ***(6 pts)***

1. Prove that the HCF of 85 and 119 is 17. (write down your calculations)
2. Write the fraction $\frac{85}{119}$ in its simplest form. (justify)
3. A maestro organizes a rehearsal with 204 members of a chorus, for a concert. There are 85 males and 119 females. He wants to form rehearsal groups such that:
   - Each group contains the same number of males and females.
   - The number of males is the same in each group.
   - Every member of the chorus belongs to one group.

   What is the maximum number of groups he can form and how many males and females will then be in each group ? (justify)

Exercice 2. ***(9 pts)***

A family is studying two possible tariffs for its electricity supply:

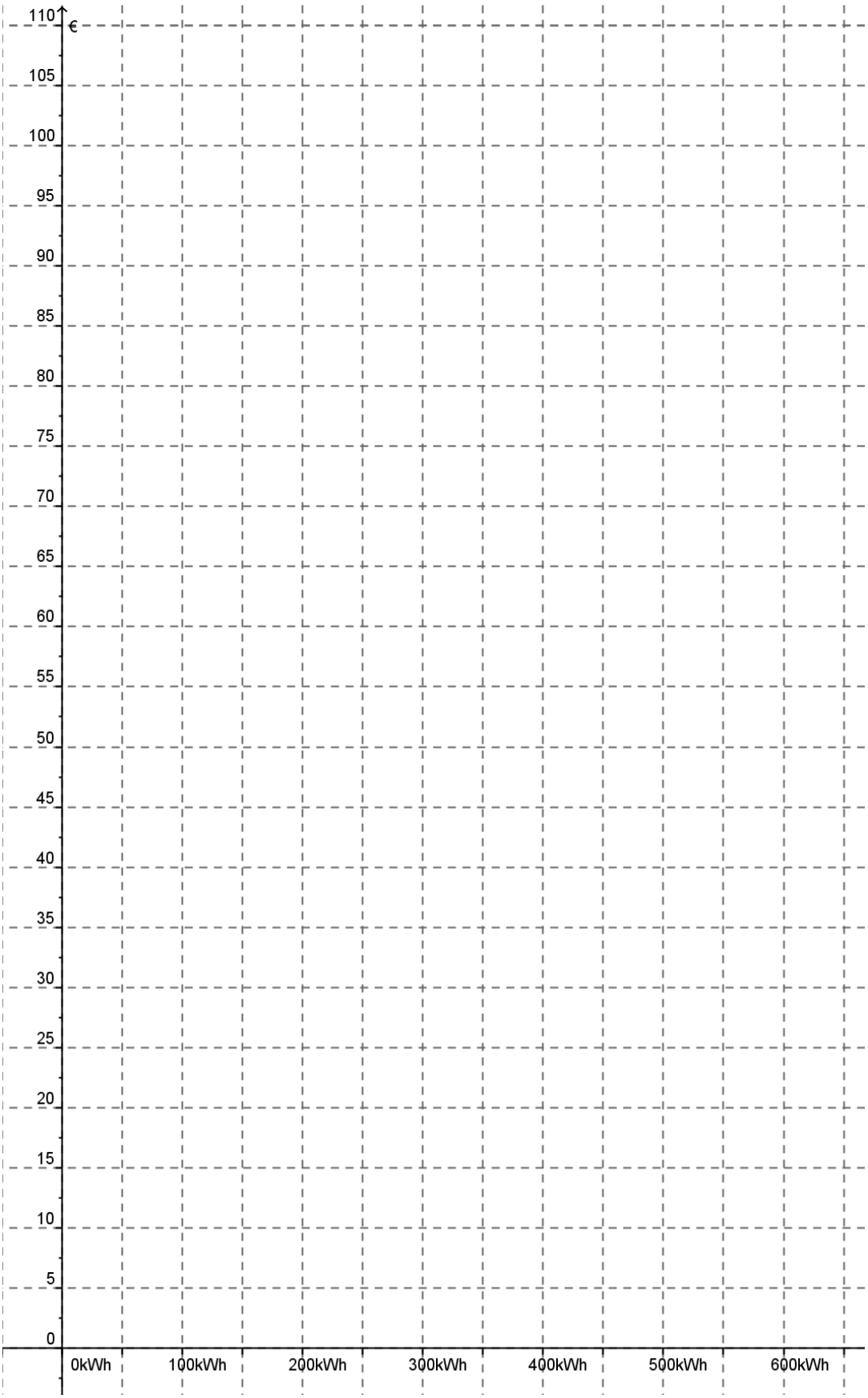
	Tariff 1	Tariff 2
Monthly subscription	0	36
Price per kWh	0.24	0.14

Part 1

1. If the family consumes 300 kWh in a month, calculate the cost with tariff 1 and then for tariff 2.
2. If the family consumes 450 kWh in a month, calculate the cost with tariff 1 and then for tariff 2.
3. Knowing that the family has paid 112.8€ in one month with tariff 1, work out its consumption in kWh.
4. Let x be the number of kWh consumed, and $T_1(x)$ and $T_2(x)$ the corresponding cost for one month with tariffs 1 and 2 respectively. We hence have $T_1(x)=0.24x$ and $T_2(x)=36+0.14x$. Find the value of x for which $T_1(x)=T_2(x)$.

Part 2

1. On the graph below, the electricity consumption is represented on the x-axis (1 cm for 50 kWh) and the cost on the y-axis (1 cm for 5 €).
   a. Draw the line (d_1) representing the function T_1.
   b. Draw the line (d_2) representing the function T_2.
2. a. From the graph, find the cost of a consumption of 400 kWh with tariff 1.
   b. From the graph, find the consumption for a cost of 106€ with tariff 2.
3. From the graph, find the cheaper tariff depending on the consumption of the family.



THE END

Numerical Activities

Exercice 1. ***(3 pts)***

1. The price after a discount of 25% is $120\times0.75=90€$ (Multiplier 1-0.25=0.75)
2. The initial price was $78.4\div0.80=98€$ (Multiplier 1-0.20=0.8)
3. The discount on the golden watch is $\frac{210-280}{280}=-0.25=-25\%$

Exercice 2. ***(4 pts)***

Consider : $D=9x^2-4+(3x-2)(x-3)$.

1. $D=9x^2-4+3x^2-2x-9x+6=12x^2-11x+2$.
2. $9x^2-4=(3x-2)(3x+2)$ thus $D=(3x-2)\left[(3x+2)+(x-3)\right]=(3x-2)(4x-1)$.
3. For $x=-5$, $D=(-15-2)(-20-1)=357$.
4. $(3x-2)(4x-1)=0$ if and only if, either $(3x-2)=0$ or $(4x-1)=0$. The solutions are thus $x=\frac{2}{3}$ and $x=\frac{1}{4}$.

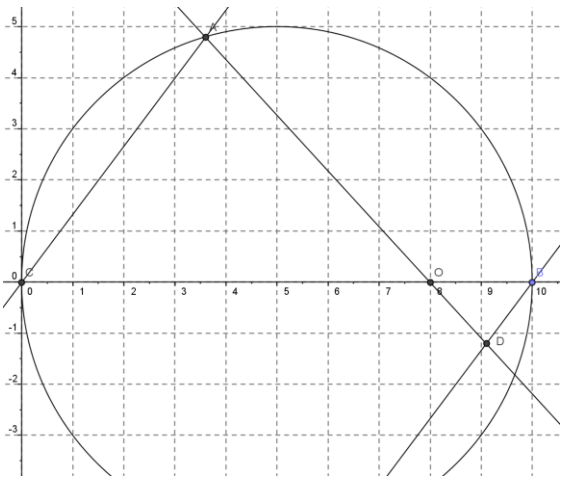
Exercice 3. ***(3 pts)***

1. $B = 4\sqrt{7} - 8\sqrt{28} + \sqrt{700} = 4\sqrt{7} - 8\times2\sqrt{7} + 10\sqrt{7} = -2\sqrt{7}$.

2. $C=\left(4\sqrt{5}+2\right)^2=16\times5+2\times2\times4\sqrt{5}+4=84+16\sqrt{5}$.

GEOMETRICAL ACTIVITIES

Exercice 1. ***(6 pts)***



1. ABC is right-angled thanks to the circle theorem ([AB] being a diameter]).
2. Thanks to the Pythagoras' theorem : $AC^2=BC^2-AB^2=100-64=36$. Thus $AC=\sqrt{36}=6$
3. In the right-angled triangle ABC we have : $\cos ABC=\frac{AB}{BC}=\frac{8}{10}=0.8$. Using the calculator : $ABC=\cos^{-1}(0.8)\approx37°$.
4. Since (AC) and (BD) are parallel, the triangles OAC and ODB form a Thalès configuration : $\frac{AC}{BD}=\frac{OC}{OB}$, thus $\frac{AC\times OB}{OC}=BD$, thus $BD=\frac{6\times2}{8}=1.5$

Exercice 2. ***(6 pts)***

1. $BCA = 180° - (90° + 10°) = 80°$.

2. In the right-angled triangle ABC we have : $\tan ABC = \tan 10° = \frac{AC}{AB}$. Thus $AC = AB\tan 10° \approx 500 \times 0.1763 \approx 88m$.

3. In the right-angled triangle ABC we have : $\cos ABC = \cos 10° = \frac{AB}{BC}$. Thus $BC = \frac{AB}{\cos 10°} \approx 508m$.

4. In the right-angled triangle HBD we have : $\cos HBD = \cos 10° = \frac{BH}{BD}$. Thus $BD = \frac{BH}{\cos 10°} \approx 406m$.

Problems

Exercice 1. ***(6 pts)***

1. Last non-zero remainder is 17 using the Euclid's algorithm :

Euclid's algorithm

A	B	remainder
119	85	34
85	34	17
34	17	0

2. $\frac{85}{119} = \frac{\cancel{17} \times 5}{\cancel{17} \times 7} = \frac{5}{7}$. It is the simplest form since 5 and 7 are coprimes.

3. He will be able to form 17 groups with 5 males and 7 females in each group ($17 \times 5 = 85$ and $17 \times 7 = 119$)

Exercice 2. ***(9 pts)***

Part 1

1.For 300 kWh in a month, the cost with tariff 1 is $0.24 \times 300 = 72€$ and $36 + 0.14 \times 300 = 78€$ with tariff 2.

2.For 450 kWh in a month the cost with tariff 1 is $0.24 \times 450 = 108€$ and $36 + 0.14 \times 450 = 99€$ with tariff 2.

3. $0.24 \times x = 112.8€$, thus its consumption in kWh is $x = \frac{112.8}{0.24} = 470$.

4. $0.24x = 36 + 0.14x$ thus $0.24x - 0.14x = 36$ or $0.10x = 36$, thus the value of x for which $T_1(x) = T_2(x)$ is 360.

Part 2

1.See opposite.

2.a. The cost of a consumption of 400 kWh with tariff 1 : 96€ (point B)

b. The consumption for a cost of 106€ with tariff 2 : 450 kWh (point C).

3.Up to 360 kWh, tariff 1 is cheaper and from 360 kWh onwards tariff 2 is more interesting.