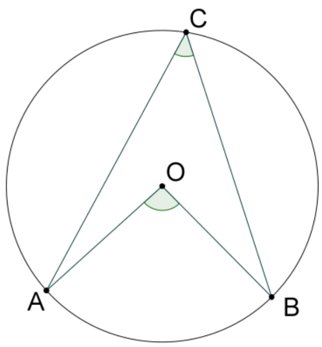
CIRCLE THEOREMS AND REGULAR POLYGONS

## 1. Angle at the circumference, angle at the centre

In this part, we will be using a ***circle*** centre O.

### 1.1 Vocabulary




A, B and C are three points on the circle.

**$\widehat{ABC}$ is an angle subtended by the arc $\overset{\frown}{AB}$ at the circumference**

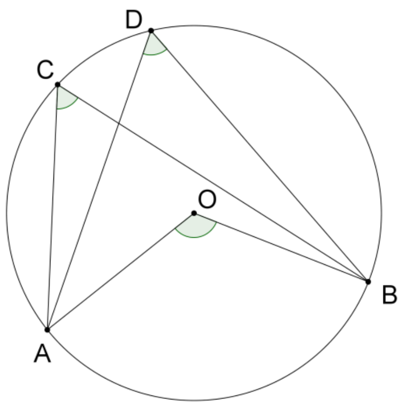
*(Fr : un angle inscrit qui intercepte l'arc* $\overset{\frown}{AB}$ *. )*

**$\widehat{AOB}$ is the angle subtended by the arc $\overset{\frown}{AB}$ at the centre of the circle**

*(Fr: l'angle du centre qui intercepte l'arc* $\overset{\frown}{AB}$ ).

Note : There is only one angle subtended by the arc $\overset{\frown}{AB}$, at the centre of the circle, but there are many different angles subtended at the circumference.

### 1.2 Property




On the diagram :

$\widehat{AOB}$ = ......................

$\widehat{ACB}$ = ......................

$\widehat{ADB}$ = ......................

***Angle at the centre is twice the angle formed at the circumference***

***Consequence :***
***All the angles subtended at the circumference by the same arc ……..………………………………………………………..***

(Or: angles in the same segment are equal)

## 2. Regular polygons

### 2.1 Definition

***A regular polygon (Fr : polygone régulier) is a polygon whose sides are the same length and whose angles are the same size.***

Examples

### 2.2 Properties

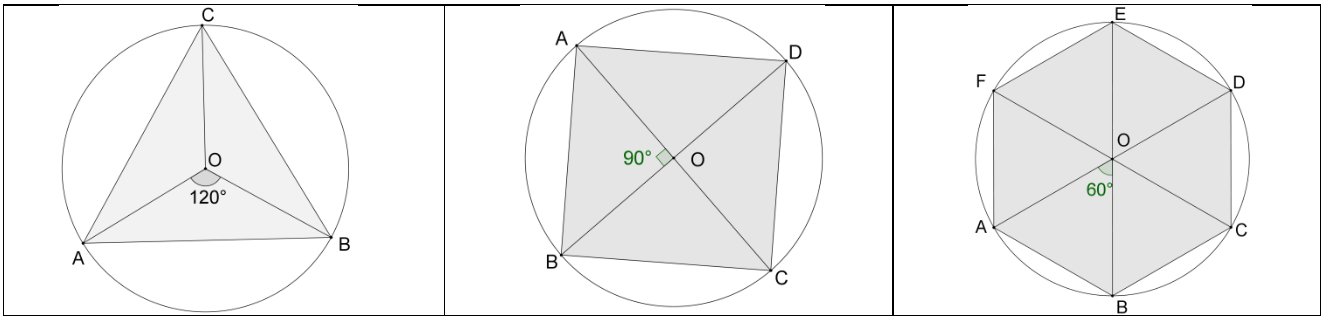
> ***All regular polygons are cyclic (Fr : inscrits dans un cercle) : there exists a circle which goes through all the vertices of a regular polygon. The centre of the circle is called the centre of the regular polygon.***

For an equilateral triangle, this centre is the ……………………………………………………………………………………..

For the square and the hexagon, the centre is ……………………………………………………………………………………..

> ***Property :***
>
> ***In an n-sided regular polygon of centre O. If [AB] is one of its sides, then*** $AOB = \frac{360}{n}$

<u>Examples</u>




**<u>4) Method for constructing an equilateral triangle, a square and a regular hexagon</u>**

<table>
<tr><th></th><th>Equilateral triangle</th><th>Square</th><th>Regular hexagon</th></tr>
<tr><td rowspan="4">With centre O and a vertex A :</td><td colspan="3">1) Draw the circle centre O and radius OA.</td></tr>
<tr><td colspan="2">2) Mark the point B on the circle such that $AOB$ is :</td><td rowspan="3">2) Set the compass to length OA. Go round the circle from A, marking off the vertices until you get back to A.</td></tr>
<tr><td>120°</td><td>90°</td></tr>
<tr><td colspan="2">3) Using a compass, set it to length AB then, starting from B, mark point(s) on the circle at distance AB from each other until you get back to A. Join up the vertices.</td></tr>
<tr><td>With side [AB] :</td><td>Make two arcs of circles radius AB with centre A and B respectively.<br>The intersection is point C, the third vertex of the triangle.</td><td>Draw the lines perpendicular to (AB) which go through A and B respectively.<br>Using a compass, mark length AB on these lines to obtain the two other vertices C and D</td><td>Draw the equilateral triangle AOB. O is the centre of the regular hexagon. Draw in the circle centre O, radius OA then follow the preceding method.</td></tr>
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