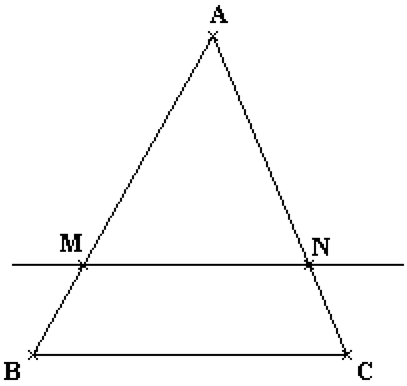
Thales' theorem

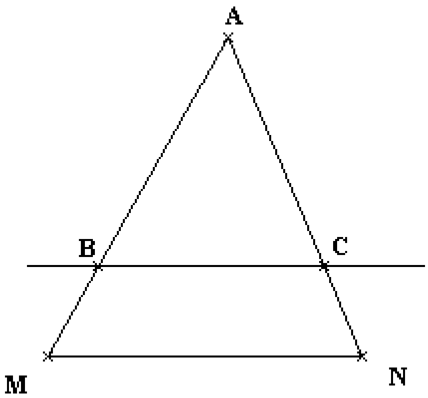
1. Thales' theorem:

> Two intersecting lines cut by two parallel lines make two triangles. The lengths of whose sides are proportional.

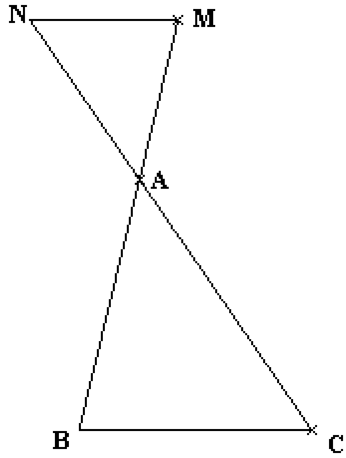
There are three Thales configurations :



(MN) // (BC)



(MN) // (BC)



(MN) // (BC)

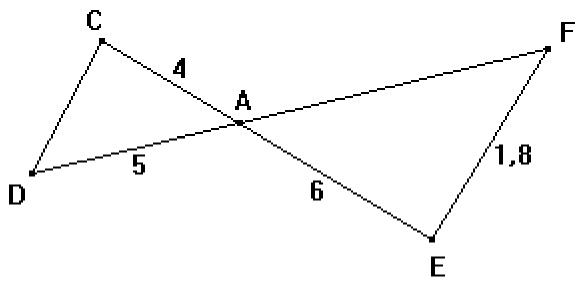
(AB) and (AC) are two lines that intersect at A, and (MN) is parallel to (BC).

So, by Thales' Theorem, $\frac{\mathbf{AM}}{\mathbf{AB}} = \frac{\mathbf{AN}}{\mathbf{AC}} = \frac{\mathbf{MN}}{\mathbf{BC}}$

2. Applications of Thales

2.1 Calculating lengths

The diagram opposite is not to scale. The unit is the cm.
Given that (CD)//(FE), calculate AF and CD



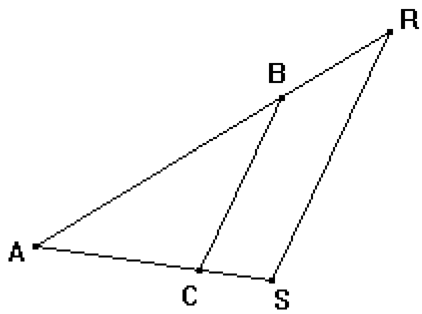
(CE) and (DF) intersect at A and (CD) is parallel to (EF)

So, by Thales' Theorem: $\frac{\mathbf{AC}}{\mathbf{AE}} = \frac{\mathbf{AD}}{\mathbf{AF}} = \frac{\mathbf{CD}}{\mathbf{EF}}$ $\left(\frac{\textbf{small triangle}}{\textbf{big triangle}}\right)$

We have $\frac{4}{6} = \frac{5}{AF} = \frac{CD}{1.8}$ so $\frac{4}{6} = \frac{5}{AF}$ which gives us
AF=5x6/4=7.5 <u>AF=7.5cm</u>
And $\frac{4}{6} = \frac{CD}{1.8}$ which gives us CD=4x1.8/6=1.2 <u>CD=1.2cm</u>

2.2 Proving that two lines are not parallel

Consider the diagram opposite. The unit is the cm.
We have:
AC = 4 AS = 5.3 AB = 9 BR = 3.
Prove that the lines (BC) and (RS) are not parallel.



(CS) and (BR) intersect at A

We have $\frac{AC}{AS} = \frac{4}{5.3} = 0.7547$

and $\frac{AB}{AR} = \frac{9}{12} = 0.75$

$\frac{AC}{AS} \neq \frac{AB}{AR}$ So, by Thales' Theorem (BC) and (RS) are not parallel

3. Converse (Fr : réciproque) of Thales' Theorem

If $\frac{AM}{AB} = \frac{AN}{AC}$ and if the points A, B, M and the points A, C, N are aligned in the same order, then the lines (BC) and (MN) are parallel.

Example:

ABC is a triangle with AB=8cm, AC=6cm and BC=4cm.

M and N are points on [AB] and [AC] respectively such that AM=6cm and AN=4.5cm.

Prove that (BC) is parallel to (MN).

*First calculate the two quotients and compare them by giving them the same denominator, or writing as an **exact** decimal*

On the one hand : $\frac{AM}{AB} = \frac{8}{5} = \frac{24}{15}$

On the other hand: $\frac{AN}{AC} = \frac{12}{7.5} = \frac{24}{15}$

So $\frac{AM}{AB} = \frac{AN}{AC}$.

Also, the points A, N, C and A, M B are aligned in the same order, so by the converse of Thales' Theorem, (NM)//(NB)

Special case (Mid point theorem)

In the triangle ABC, M is the mid point of [AB] and N is the mid point of [AC].

The points A, M, B and A, N, C are in the same order and $\frac{AM}{AB} = \frac{AN}{AC} = \frac{1}{2}$

So by the converse of Thales, (MN) and (BC) are parallel