TLE S TEST N°3

FRIDAY OCTOBER 26TH 2012

duration : 1h

Ecole Internationale PACA à Manosque Tle S

EXERCICE 1. (15 PTS) FROM BAC S NEW CALEDONIA MARCH 2011

Every year, two villages A and B organise a sports event. The contestants have to randomly pick a means of transport to get from A to B, as fast as they can, on an appropriate path.
For the draw, an urn containing 4 identical tokens is used. The tokens are labeled B, R, F and L.
A contestant randomly picks a token :

- if it's labelled B, he will have to use a bike
- if it's labelled R, he will have to use roller skates
- if it's labelled F, he will have to go on foot
- if it's labelled L, he can freely choose one of the three previous options.

From the previous years, we know that, when a contestant picks the L token, he chooses a bike 7 times out of 10, roller skates 2 times out of 10 and to go by foot one time out of 10.

1. Draw up a weighted probability tree diagram representing this situation.

   For the following questions, the results have to be given rounded to 0.001

2. What is the probability for a contestant to go on foot given that he has picked the token L ?

3. Work out the probability for a contestant to use roller skates and pick the L token.

4. Work out the probability for a contestant to use a bike.

5. Knowing that a contestant has used a bike, what is the probability he had picked the L token ?

6. Are the events "he has picked the L token" and "he uses a bike" independent ?

7. We assume that the results from one year to the next are independent. From the results of previous years, we know that the probability for the winner to have used a bike is 2/3.
   Calculate the probability that, in the 6 coming years, the race is won at least once by a non-biker. (you can draw up a probability tree diagram)

EXERCICE 2. (5 PTS)

"Selected Review of Knowledge Covered" (fr : Restitution Organisée de Connaissance)

Prove that, if A and B are independent events, then it's also the case for $\overline{A}$ and B.

BONUS POINTS ON MATH CULTURE (2.5 PTS)

(On the BBC4 podacst "Carl Friedrich GAUSS" by Markus du Sautoy)

Answer with a single sentence

1. What event has made Gauss become famous ? When did it take place and how old was he ?
2. In which field of mathematics did he first work ?
3. Who has helped Gauss to have access to university and became his mentor ?
4. What is his main achievement ? Could you sketch up a Gaussian curve ?
5. What is the sentence which is known as Gauss' most famous quotation ?

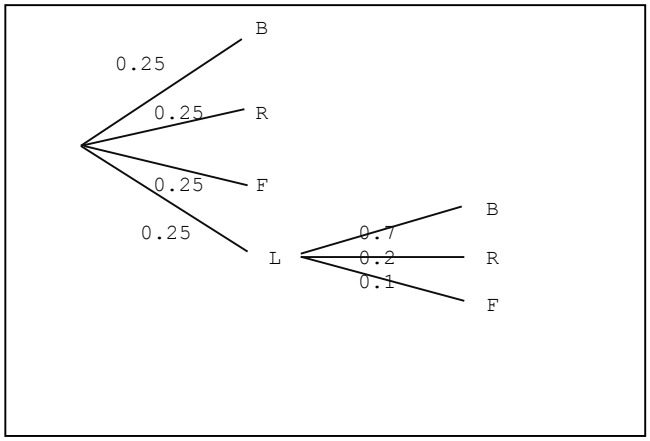
Nice Holidays !

TEST n°3 Correction

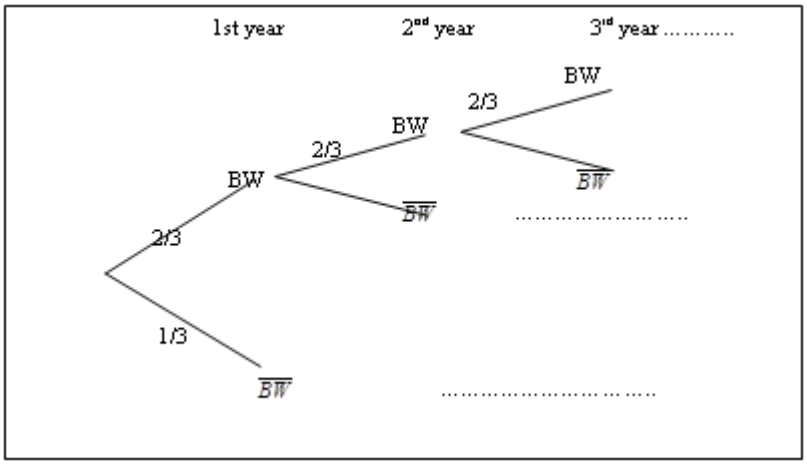
Ecole Internationale PACA à Manosque Tle S

EXERCICE 1. (15 PTS)

1. .
2. $P_L(F) = 0.1$
3. $P(L \cap R) = 0.25 \times 0.2 = 0.05$
4. $P(B) = 0.25 + 0.25 \times 0.7 = 0.425$
5. $P_B(L) = \dfrac{P(B \cap L)}{P(B)} = \dfrac{0.25 \times 0.7}{0.425} \approx 0.412$



6. $P_B(L) \neq P(L)$, hence L and B are not independent.
7. To calculate the probability that, in the 6 coming years, the race is won at least once by a non-biker, we can consider the following tree :



Let's denote BW the event "the race is won by a biker", N the event "in the 6 coming years, the race is won at least once by a non-biker" is the complementary of M : "in the 6 coming years, the race is always won by a biker"

$P(M) = \left(\dfrac{2}{3}\right)^6$, so

$P(N) = 1 - \left(\dfrac{2}{3}\right)^6 = 0.912$

EXERCICE 2. (5 PTS)

The events $A \cap B$ and $\overline{A} \cap B$ form a partition of B. Hence $P(A \cap B) + P(\overline{A} \cap B) = P(B)$, but A and B being independent, $P(A \cap B) = P(A) \times P(B)$, so $P(\overline{A} \cap B) = P(B) - P(A \cap B) = P(B) - (P(A) \times P(B)) = P(B) \times (1 - P(A)) = P(B) \times P(\overline{A})$, which proves that $\overline{A}$ and B are independent.

BONUS POINTS ON MATH CULTURE (2.5 PTS)

(On the BBC4 podacst "Carl Friedrich GAUSS" by Markus du Sautoy)

1. In 1801, aged only 24, he was able to predict to position of Cérès, an asteroid (first thought as a new planet) discovered a few months before but had then disappeared again.
2. Statistics
3. Duke Ferdinand of Brunswick
4. Having found the appearance of regular patterns in data series and specially the way they are distributed.
5. "Mathematics is the queen of sciences"