TLE S TEST N°5
MONDAY JANUARY 28TH 2013

EXERCISE 1: (8 PTS)

Let us consider the following complex numbers : $z_1 = \sqrt{2} + i\sqrt{6}$, $z_2 = 2 + 2i$ and $Z = \frac{z_1}{z_2}$.

1) Give the algebraic form of Z.
2) Give the polar form of z_1, z_2, Z.
3) Deduce the exact value of $\cos\frac{\pi}{12}$ and $\sin\frac{\pi}{12}$.
4) Work out the algebraic form of Z^{2015} .

EXERCISE 2: (4 PTS)

In the complex plane, we consider the following points A, B and C associated with the complex numbers: $a = 1$, $b = 1 + 2i$ and $c = 1 + \sqrt{3} + i$ respectively.

1. Give the exponential form of $\frac{c-a}{b-a}$.
2. What kind of triangle is ABC ?

EXERCICE 3. (5 PTS) FROM BAC S MCQ

For each question, only one answer is correct. On your paper, write the number of the question and the letter of the answer. No justification is necessary.

A correct answer gives 1 point, a false answer takes 0.5 points off and no answer gives 0 points. At the end of the exercise, if the total of points is negative, then the grade will be equal to 0.

1) Let z be a complex number. $|z+i|$ is equal to: a) $|z|+1$ b) $|z-1|$ c) $|i\bar{z}+1|$

2) Let z be a non-zero complex number with argument θ. An argument of $\frac{-1+i\sqrt{3}}{\bar{z}}$ is:

a) $-\frac{\pi}{3}+\theta$ b) $\frac{2\pi}{3}+\theta$ c) $\frac{2\pi}{3}-\theta$

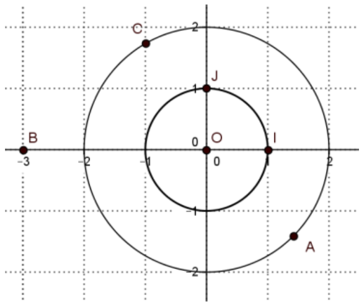
3) Let n be a natural number. The complex number $(\sqrt{3}+i)^n$ is purely imaginary if and only if:
a) $n = 3$ b) $n = 6k+3$ with $k \in \mathbb{Z}$ c) $n = 6k$ with $k \in \mathbb{Z}$

4) Let A and B be two points associated with the complex numbers **i** and **-1** respectively. The locus of the points M associated with the complex number z so that $|z-i| = |z+1|$ is: a) the line (AB)
b) the circle of diameter [AB] c) the perpendicular line to (AB) through O

5) The exponential form of the complex number $\frac{\sqrt{2}}{1-i}$ is : a) $e^{i\frac{\pi}{4}}$ b) $e^{-i\frac{\pi}{2}}$ c) $\sqrt{2}e^{-i\frac{\pi}{4}}$.

EXERCICE 4. (3 PTS)

Give both the exponential and Cartesian forms of the complex number associated with the points A, B and C opposite.



BONUS POINTS ON MATH CULTURE (2 PTS)

(On the document "Introduction à l'étude des nombres complexes" by Anne Boyé (IREM))

1. In Which kind of problem where what we call now complex numbers introduced ?
2. When did it roughly take place ? Where ?
3. Quote one mathematician involved in this breakthrough.
4. Who first introduced the notation i ? what was the problem of using $\sqrt{-1}$?

EXERCISE 1: (8 PTS)

1) $Z=\dfrac{\sqrt{2}+i\sqrt{6}}{2+2i}=\dfrac{\left(\sqrt{2}+i\sqrt{6}\right)(2-2i)}{(2+2i)(2-2i)}=...=\dfrac{\sqrt{2}+\sqrt{6}}{4}+i\dfrac{\sqrt{6}-\sqrt{2}}{4}$.

2) $|z_1|=\sqrt{2+6}=2\sqrt{2}$ and $\cos(\arg(z_1))=\dfrac{\sqrt{2}}{2\sqrt{2}}=0.5$ and $\sin(\arg(z_1))=\dfrac{\sqrt{6}}{2\sqrt{2}}=\dfrac{\sqrt{3}}{2}$, so $\arg(z_1)=\dfrac{\pi}{3}$, and hence $z_1=2\sqrt{2}\left(\cos\left(\dfrac{\pi}{3}\right)+i\sin\left(\dfrac{\pi}{3}\right)\right)$. Idem $z_2=2\sqrt{2}\left(\cos\left(\dfrac{\pi}{4}\right)+i\sin\left(\dfrac{\pi}{4}\right)\right)$:

Therefore $|Z|=\dfrac{|z_1|}{|z_2|}=1$ and $\arg(Z)=\arg(z_1)-\arg(z_2)=\dfrac{\pi}{12}$, so $Z=\cos\left(\dfrac{\pi}{12}\right)+i\sin\left(\dfrac{\pi}{12}\right)$

3) Using both forms of Z, we get $\cos\left(\dfrac{\pi}{12}\right)=\dfrac{\sqrt{2}+\sqrt{6}}{4}$ and $\sin\left(\dfrac{\pi}{12}\right)=\dfrac{\sqrt{6}-\sqrt{2}}{4}$.

4) $|Z^{2015}|=|Z|^{2015}=1^{2015}=1$ and $\arg\left(Z^{2015}\right)=2015.\arg(Z)=\arg\left(\dfrac{2015\pi}{12}\right)=\arg\left(84\times2\pi-\dfrac{\pi}{12}\right)=-\dfrac{\pi}{12}$. The points on the trigonometric circle for $-\dfrac{\pi}{12}$ and $\dfrac{\pi}{12}$ being symmetrical about the x-axis,

$Z^{2015}=\dfrac{\sqrt{2}+\sqrt{6}}{4}+i\dfrac{-\sqrt{6}+\sqrt{2}}{4}$.

EXERCISE 2: (4 PTS)

In the complex plane, we consider the following points A, B and C associated with the complex numbers: $a=1$, $b=1+2i$ and $c=1+\sqrt{3}+i$ respectively.

$\dfrac{c-a}{b-a}=...=e^{-\frac{\pi}{3}}$, hence $\left|\dfrac{c-a}{b-a}\right|=\dfrac{AC}{AB}=1$ and $\left(\overrightarrow{AB};\overrightarrow{AC}\right)=\arg\left(\dfrac{c-a}{b-a}\right)=-\dfrac{\pi}{3}$. ABC is thus an indirect equilateral triangle.

EXERCICE 3. (5 PTS) FROM BAC S MCQ

$|z+i|$ is equal to: c) $|i\bar{z}+1|$

Let z be a non-zero complex number with argument θ. An argument of $\dfrac{-1+i\sqrt{3}}{\bar{z}}$ is: b) $\dfrac{2\pi}{3}+\theta$

The complex number $(\sqrt{3}+i)^n$ is purely imaginary if and only if: b) $n=6k+3$ with $k\in\mathbb{Z}$

The locus of the points M associated with the complex number z so that $|z-i|=|z+1|$ is:

c) the perpendicular line to (AB) through O

The exponential form of the complex number $\dfrac{\sqrt{2}}{1-i}$ is $e^{i\frac{\pi}{4}}$.

EXERCICE 4. (3 PTS)

$z_A=2e^{-i\frac{\pi}{4}}=\sqrt{2}-i\sqrt{2}$ $z_B=3e^{i\pi}=-3$ $z_C=2e^{i\frac{2\pi}{3}}=-1+i\sqrt{3}$

BONUS POINTS ON MATH CULTURE (2 PTS)

1. Solving 3rd degree equations
2. In Italy, during the XVIth century ?
3. Tartaglia, Cardan, Bombelli , …..
4. Euler, in 1777. $\left(\sqrt{-1}\right)^2=-1$ (by definition) and $\left(\sqrt{-1}\right)^2=\sqrt{-1}\times\sqrt{-1}=\sqrt{(-1)\times(-1)}=\sqrt{1}=1$