TLE S TEST N°6
FRIDAY FEBRUARY 15TH 2013

Ecole Internationale PACA à Manosque Tle S

EXERCISE 1

1. Simplify $A = \ln 216 - 3(\ln 2 + \ln 3)$

2. Solve in $\mathbb{R}$: $\ln x + \ln(x-3) = \ln 4$

EXERCISE 2

We assume that $\lim_{x \to 0} (x \ln x) = 0$

We consider the function f defined on $]0;+\infty[$ by : $f(x) = 1 - \dfrac{\ln x}{x+1}$. Let Γ be its graph.

1. Study of an auxiliary function

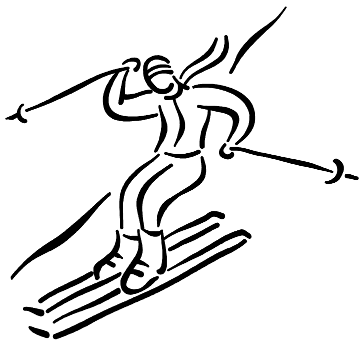
Let g be the function defined on $]0;+\infty[$ by : $g(x) = x \ln x - x - 1$

a. Study the variations of *g*.
b. Find its limits at 0 and $+\infty$.
c. Justify that the equation *g(x) = 0* has a unique solution α in $]0;+\infty[$ .Give an approximate value of α (to 10^{-2}).
d. Deduce then the sign of *g(x)* with regards to *x*.

2. Study of f

a. Prove that the derivative f' has the same sign as *g*.
b. Deduce then the table of variations of f.
c. Draw up the graph Γ.

HAPPY HOLYDAYS !



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CORRECTION

EXERCISE 1

1. $\ln 216 = \ln\left(6^3\right) = 3\ln\left(2\times 3\right) = 3\left(\ln 2 + \ln 3\right)$ hence $A = 0$

2. (E) $\ln x + \ln\left(x-3\right) = \ln 4$ Existing conditions : $\left(x > 0 \text{ and } x > 3\right)$ so $x > 3$

on $I = \left]3;+\infty\right[$, $\left(E\right) \Leftrightarrow \ln\left(x\left(x-3\right)\right) = \ln 4 \Leftrightarrow x\left(x-3\right) = 4$ since the ln function is strictly increasing on I. $x\left(x-3\right) = 4 \Leftrightarrow x^2 - 3x - 4 = 0 \Leftrightarrow .... \Leftrightarrow x = 4 \text{ or } x = -1$

<u>So, according to the existing conditions, the only solution of (E) is 4</u>

EXERCISE 2

1a. $g(x) = x\ln x - x - 1$

$g'(x) = 1\times\ln x + x\times\frac{1}{x} - 1 = \ln x$ thus g is decreasing on $\left]0;1\right]$ and increasing on $\left[1;+\infty\right[$

1b. * $\lim_{x\to 0} g(x) = -1$ since $\lim_{x\to 0}(x\ln x) = 0$

* $g(x) = x\ln x - x - 1 = x\left(\ln x - 1 - \frac{1}{x}\right)$ *thus* ...... $\lim_{x\to+\infty} g(x) = +\infty$

1c. Table of variation of g:

x	0		1		α		$+\infty$
$g'(x)$	‖	$-$	0	$+$			
$g(x)$	‖ -1	↘	-2	↗		↗	$+\infty$
		$-$		$-$	0	$+$	

* g is continuous and decreasing on $\left]0;1\right]$, thus, if $0 < x \le 1$, then $g(x) < \lim_{x\to 0} g(x)$, hence $g(x) < 0$, hence $g(x) = 0$ has no solution in $\left]0;1\right]$.

* g is continuous and increasing on $\left[1;+\infty\right[$, $g(x)$ belonging to $\left[-2;+\infty\right[$. $0 \in \left[-2;+\infty\right[$, hence, according to the intermediate value theorem, the equation $g(x) = 0$ has a unique solution α in $\left[1;+\infty\right[$.

* Finally, the equation $g(x) = 0$ has a unique solution α in $\left]0;+\infty\right[$.

* With the calculator, we get $\alpha \approx 3{,}59$

2a. $f'(x) = 0 - \dfrac{\frac{1}{x}\times(x+1) - \ln x\times 1}{(x+1)^2} = \dfrac{-1-\frac{1}{x}+\ln x}{(x+1)^2} = \dfrac{-x-1+x\ln x}{x(x+1)^2} = \dfrac{g(x)}{x(x+1)^2}$

$x(x+1)^2$ being positive on the domain of *f*, $f'(x)$ has the same sign as $g(x)$.

2b.

x	0		α		$+\infty$
$f'(x)$	‖	-	0	+	
$f(x)$	‖ $+\infty$	↘	≈ 0.72	↗	1

2c.