TLE S   TEST N°7

FRIDAY MARCH 8TH 2013

Ecole Internationale PACA à Manosque Tle S

EXERCISE 1. (8 PTS)

1. Calculate the following integrals :

a) $\int_1^2 \frac{2x+1}{x^2+x-1}dx$ b) $\int_{-\ln 2}^{0} e^{2x}-e^x+1\,dx$

2. Calculate the mean value of the function :

a) $f(x)=\sin\left(\frac{x}{2}+\pi\right)$ on $[0 ; \pi]$ b) $f(x)=x^2$ on $[-1 ; 1]$

EXERCICE 2. (12 PTS) BACCALAURÉAT S ANTILLES SEPTEMBRE 2011

We consider the function f, defined on $]0;+\infty[$ by : $f(x)=x\ln x-1$.

Part A : Study of a function.

1. a. Work out the limit of f at $+\infty$.
   b. Work out the limit of f at 0.

2. Let f' be the derivative of the function f. Calculate $f'(x)$ for any x in $]0;+\infty[$. Deduce the table of variation of f on $]0;+\infty[$.

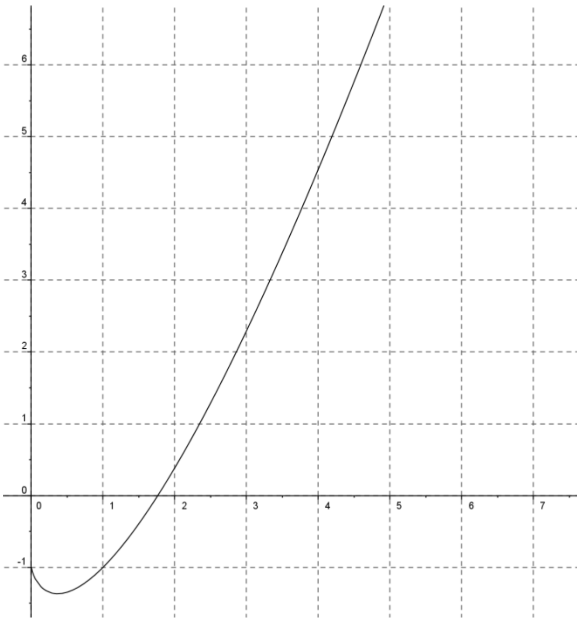
3. Prove that the equation $f(x)=0$ has a single solution in $]0;+\infty[$. We denote α this solution. Give an approximate value of α rounded to 0.01 .

4. Find the sign of $f(x)$ when x belongs to $]0;+\infty[$.

5. Prove that $\ln\alpha=\frac{1}{\alpha}$.

Part B : Calculation of an integral.

Opposite is the graph of f . We consider $I=\int_{\alpha}^{4} f(x)\,dx$.



1. Justify that I is the value of the area of specific domain in the plane that you will stripe (don't forget to hand-in this sheet)

2. Prove that the function $x \mapsto g(x)=\frac{1}{4}x^2(2\ln x-1)$ is a primitive of $x \mapsto x\ln x$ on $]0;+\infty[$.

3. Prove that $I=\frac{\alpha^2}{4}+\frac{\alpha}{2}+16\ln 2-8$. Deduce then an approximate value of I rounded to 1 dp.

BONUS POINTS ON MATH CULTURE (2.5 PTS)

(on the BBC 4 podcast "Joseph Fourier" by Markus du Sautoy)

<u>**Answer with a single sentence**</u>

1. What was the main branch of maths Fourier was interested in at the begining ?
2. Which event allowed him to make studies and become a teacher ? at which school ?
3. What was his main achievement in the field of sound theory ?

EXERCISE 1

Done in class

EXERCICE 2. (5 PTS) BACCALAUREAT S ANTILLES SEPTEMBRE 2011

1. a. by product $\lim\limits_{x\to+\infty} f(x)=+\infty$, since $\lim\limits_{x\to+\infty} x=+\infty$ and $\lim\limits_{x\to+\infty} \ln x=+\infty$.

   b. $\lim\limits_{x\to 0^+} f(x)=-1$ since $\lim\limits_{x\to 0^+} x\ln x=0$

2. $f'(x)=(uv)'=u'v+v'u=\ln x+x\times\frac{1}{x}=\ln x+1$ for any x in $]0;+\infty[$, with u = …, v= ….. $\ln x+1>0 \Leftrightarrow \ln x>-1 \Leftrightarrow x>e^{-1}$ since $x\mapsto e^x$ is strictly increasing. Hence the table of variation of f on $]0;+\infty[$.

x	0		e^{-1}		$+\infty$
$f'(x)$		$-$	0	$+$	
$f(x)$	-1	↘	$-e^{-1}-1$	↗	$+\infty$

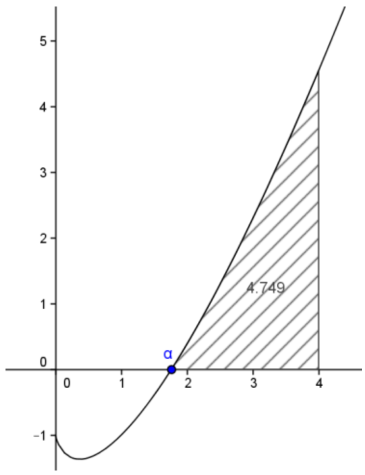
2. on $]0;e^{-1}]$, $f(x)<-1<0$ hence $f(x)=0$ has no solution.

3. on $]e^{-1};+\infty[$ f is continuous and strictly increasing. Furthermore, $f(e^{-1})<0$ and $\lim\limits_{x\to+\infty} f(x)=+\infty>0$ hence, according to the IVT, $f(x)=0$ has a unique solution α. Thanks to the calculator : $\alpha\approx 1.76$

4. According to the previous questions : if $x<\alpha$, then $f(x)<0$ and : if $x>\alpha$, then $f(x)>0$

5. $f(\alpha)=0 \Leftrightarrow \alpha\ln\alpha-1=0 \Leftrightarrow \ln\alpha=\frac{1}{\alpha}$.

Part B : Calculation of an integral.



1. On $[\alpha;4]$, f is strictly positive, hence I represents the area of the domain bounded by the graph, the x axis and the vertical lines x=0 and x=4.

2. $g'(x)=k(uv)'=\frac{1}{4}\left[2x(2\ln x-1)+x^2\frac{2}{x}\right]=\frac{1}{4}[4x\ln x-2x+2x]=f(x)$ is a primitive of $x\mapsto x\ln x$ on $]0;+\infty[$.

3. $I=\int_\alpha^4 f(x)dx=\left[\frac{1}{4}x^2(2\ln x-1)-x\right]_\alpha^4=\left(\frac{1}{4}16(2\ln 4-1)-4-\frac{1}{4}\alpha^2(2\ln\alpha-1)-\alpha\right)=$

   $=\left(\frac{1}{4}16(2\ln 4-1)-4-\frac{1}{4}\alpha^2(2\ln\alpha-1)+\alpha\right)=16\ln 2-8-\frac{1}{2}\underbrace{\alpha^2\ln\alpha}_{\alpha}+\frac{1}{4}\alpha^2+\alpha=16\ln 2-8+\frac{1}{4}\alpha^2+\frac{\alpha}{2}$

   $I\approx 4.8\,UA$.

BONUS POINTS ON MATH CULTURE (2.5 PTS)

1. The waves' theory.
2. The French revolution in 1789. The "école polytechnique".
3. Understanding that any sound can be broken up into a sum of simple sine waves.