



**STUDY AND COMPARISON OF DIFFERENT
NUMERICAL METHODS FOR SOLVING EQUATIONS
PART 1**

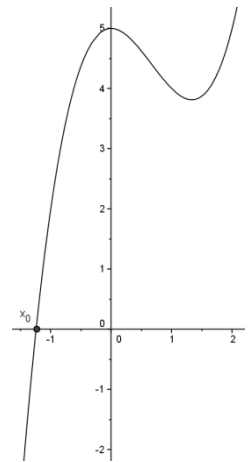


1. PRESENTATION

For most of the equations we need to solve, there is no general formula to find the exact values of the solutions (we can't solve the equation algebraically). We hence use numerical methods giving approximate values (but actually with the accuracy we need).

These methods are, for some of them, very old (since it was in fact the only way to solve equation before the birth of algebra during the century). They were of applied "by hand", but, as we will see, they can today be easily implemented in a computer or a calculator.

In the following, our problem is to find the roots of a given function f , in other words the solutions of the equation $f(x) = 0$ (which in fact allows us to solve any equation in the form $g(x) = h(x)$)



All your answer-files have to be handed in on Chamilo.

2. STUDY OF THE DIFFERENT METHODS

2.1 METHOD BY BROWSING (FR: BALAYAGE)

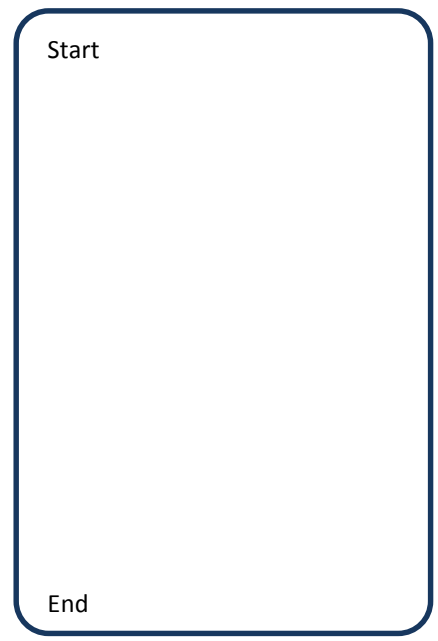
2.1.1 PRINCIPE.

Consider the equation $f(x) = 0$ with a continuous function f on the interval $[a;b]$ which takes values of different signs at the end points of the interval and which has a single root x_0 within $[a;b]$. To find x_0 approximately, one chooses a pitch h and calculates the values $f(a+h), f(a+2h), \dots$ until he observes a sign change. He then has an approximate value of the root x_0 with an accuracy less than h .

2.1.2 IMPLEMENTATION WITH AN ALGORITHM

1. Write opposite in "natural language" the algorithm associated with this method.

2. Use Scilab to program it (hand in the scinote)



2.2 METHOD BY DICHOTOMY

2.2.1 PRINCIPLE.

If is also called « method of division in halves »

Consider the equation $f(x)=0$ with a continuous function f on the interval $[a;b]$ which takes values of different signs at the end points of the interval and which has a single root x_0 within $[a;b]$.

To find x_0 approximately, one divides $[a;b]$ into halves and calculates the value of $f(x_1)$ at the midpoint $x_1 = \frac{a+b}{2}$. If $f(x_1) \neq 0$, one takes the two intervals $[a;x_1]$ and $[x_1;b]$ and from them selects for the next dichotomy the one at the end points of which the values of the function differ in sign. This continued division into halves gives a sequence x_1, x_2, \dots which converges to the root x_0 with the rate of a geometrical progression:

note :

This method is massively implemented in computers, especially search engines (we'll see why at the end).

2.2.3 IMPLEMENTATION

In this part, we will consider the function $x \mapsto f(x) = x^2 - x - 1$, since it's a quadratic and thus we can find its exact roots (we will be able to look at the convergence of our approximate roots towards the exact roots).

- ✓ Solve algebraically $f(x) = 0$: we will denote φ its positive root (it is actually the golden ratio) and φ' its negative one.

With a spread sheet

We want to get a table like the one opposite :

You're not obliged to do the absolute value bars

- ✓ make the line for $n = 0$ entering 2 values u_0 and v_0 which bound φ or φ' , and give the formulae in the circled cells. All the cells except u_0 and v_0 contain only formulae.
- ✓ How do you test whether two numbers have the same sign ?

Method by dichotomy for solving equations									
n	u_n	v_n	m	$f(m)$	$f(v_n)$	convergence			
						$ v_n - u_n $	$ m - \varphi $	$ m - \varphi' $	
0	0	5	2,5	2,75	19	5,0E+00	8,8E-01	3,1E+00	
1	0	2,5	1,25	-0,6875	2,75	2,5E+00	3,7E-01	1,9E+00	
2	1,25	2,5	1,875	0,640625	2,75	1,3E+00	2,6E-01	2,5E+00	
3	1,25	1,875	1,5625	-0,121094	0,640625	6,3E-01	5,6E-02	2,2E+00	
4	1,5625	1,875	1,71875	0,2353516	0,640625	3,1E-01	1,0E-01	2,3E+00	
5	1,5625	1,71875	1,640625	0,0510254	0,2353516	1,6E-01	2,3E-02	2,3E+00	
6	1,5625	1,640625	1,6015625	-0,03656	0,0510254	7,8E-02	1,6E-02	2,2E+00	
7	1,6015625	1,640625	1,6210938	0,0068512	0,0510254	3,9E-02	3,1E-03	2,2E+00	
8	1,6015625	1,6210938	1,6113281	-0,01495	0,0068512	2,0E-02	6,7E-03	2,2E+00	
9	1,6113281	1,6210938	1,6162109	-0,004073	0,0068512	9,8E-03	1,8E-03	2,2E+00	
10	1,6162109	1,6210938	1,6186523	0,0013831	0,0068512	4,9E-03	6,2E-04	2,2E+00	
11	1,6162109	1,6186523	1,6174316	-0,001347	0,0013831	2,4E-03	6,0E-04	2,2E+00	
12	1,6174316	1,6186523	1,618042	1,79E-05	0,0013831	1,2E-03	8,0E-06	2,2E+00	
13	1,6174316	1,618042	1,6177368	-0,000664	1,79E-05	6,1E-04	3,0E-04	2,2E+00	
14	1,6177368	1,618042	1,6178894	-0,000323	1,79E-05	3,1E-04	1,4E-04	2,2E+00	
15	1,6178894	1,618042	1,6179657	-0,000153	1,79E-05	1,5E-04	6,8E-05	2,2E+00	
16	1,6179657	1,618042	1,6180038	-6,74E-05	1,79E-05	7,6E-05	3,0E-05	2,2E+00	
17	1,6180038	1,618042	1,6180229	-2,48E-05	1,79E-05	3,8E-05	1,1E-05	2,2E+00	
18	1,6180229	1,618042	1,6180325	-3,43E-06	1,79E-05	1,9E-05	1,5E-06	2,2E+00	
19	1,6180325	1,618042	1,6180372	7,234E-06	1,79E-05	9,5E-06	3,2E-06	2,2E+00	
20	1,6180325	1,6180372	1,6180348	1,903E-06	7,234E-06	4,8E-06	8,5E-07	2,2E+00	
21	1,6180325	1,6180348	1,6180336	-7,63E-07	1,903E-06	2,4E-06	3,4E-07	2,2E+00	
22	1,6180336	1,6180348	1,6180342	5,698E-07	1,903E-06	1,2E-06	2,5E-07	2,2E+00	
23	1,6180336	1,6180342	1,6180339	-9,66E-08	5,698E-07	6,0E-07	4,3E-08	2,2E+00	
24	1,6180339	1,6180342	1,6180341	2,366E-07	5,698E-07	3,0E-07	1,1E-07	2,2E+00	
25	1,6180339	1,6180341	1,618034	7,003E-08	2,366E-07				

- ✓ Fill in the line for $n = 1$ and give the formulae in the gray cells

- ✓ you will use the function IF of the software with the test of the previous question
- ✓ Pull until $n = 25$
- ✓ Chose other values of u_0 and v_0 (negative, very large) and look at the reaction of the process.
- ✓ with $u_0 = 1$ and $v_0 = 10$ at which rank do we have an approximation to 10^{-9} ?
- ✓ Draw graphically (type « scatter plot ») the sequence (u_n) . Insert it in your answer file, along with your spreadsheet.

With an algorithm

1. Use Scilab and enter the program opposite (in a scinote)
2. Enhance the program for it to :
 - ✓ Ask for a precision h such that x_0 belongs to $[a;b]$ with $b - a < h$.
 - ✓ Display at the end the number of loops needed to reach this precision.

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dicho.sce
1 function y=f(x);y=x^2-2;
2 endfunction
3 n=input("n=");
4 a=1;b=2;
5 for k=1:n
6     c=(a+b)/2;
7     if f(a)*f(c)<0 then
8         b=c;
9     else a=c
10    end
11 disp([k,a,b])
12 end
  
```