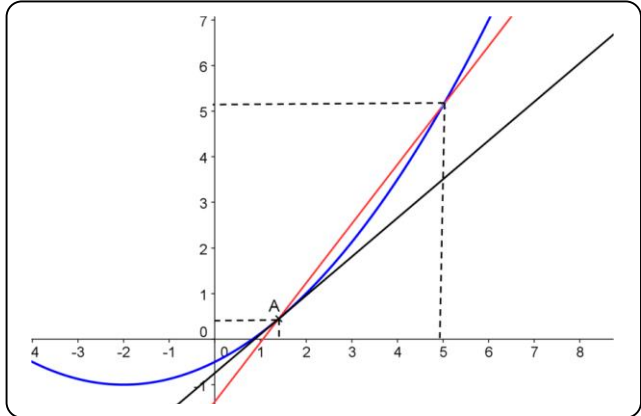




DISCOVERY OF THE EXPONENTIAL FUNCTION WITH EULER'S METHOD

1. EULER'S METHOD

Knowing $f(x_0)$ and $f'(x_0)$, the linear approximation formula : $f(x_0 + h) \approx f(x_0) + hf'(x_0)$ allows us to calculate an approximation of $f(x_0 + h)$, which is as accurate as h is small. Euler's method consists of repeating the process step by step at $f(x_0 + h)$, $f(x_0 + 2h)$, $f(x_0 + 3h)$



It allows us to sketch an approximation of the graph of a function, from only a given particular value of the function and its derivative at this point.

2. IMPLEMENTATION USING A SPREADSHEET

The exponential function is defined by $f'(x) = f(x)$ for any x in \mathbb{R} and $f(0) = 1$.
It can be denoted $f(x) = \exp(x)$

2.1 FILL IN THE FOLLOWING TABLE BY HAND :

x	0	0,1	0,2	0,3	0,4	0,5
$f'(x) = f(x)$						
$f(x)$ approximation	1					

2.2 AIM OF THE GAME :

We will implement the previous process automatically to end with a table and corresponding curve as opposite :

We'll be able to change the value of h and see the effects on the curve.

1	A	B	C	D	E	F	G	H	I
2	x	f(x)	Approximation of exp(x)						
3	value of h : 0,10								
6	-1	0,3486784401	0,3486784401						
7	-0,9	0,387420489	0,387420489						
8	-0,8	0,43046721	0,43046721						
9	-0,7	0,4782969	0,4782969						
10	-0,6	0,531441	0,531441						
11	-0,5	0,59049	0,59049						
12	-0,4	0,6561	0,6561						
13	-0,3	0,729	0,729						
14	-0,2	0,81	0,81						
15	-0,1	0,9	0,9						
16	0	1	1						
17	0,1	1,1	1,1						
18	0,2	1,21	1,21						
19	0,3	1,331	1,331						
20	0,4	1,4641	1,4641						
21	0,5	1,61051	1,61051						
22	0,6	1,771561	1,771561						
23	0,7	1,9487171	1,9487171						
24	0,8	2,14358881	2,14358881						
25	0,9	2,357947691	2,357947691						
26	1	2,5937424601	2,5937424601						

The graph shows the exponential function $f(x) = \exp(x)$ plotted from x = -1 to x = 1.1. The y-axis ranges from 0 to 3. Green square markers represent the data points from the spreadsheet, showing a smooth curve that passes through (0, 1) and increases rapidly as x increases.

2.3 YOUR TURN!

- ✓ Start with an empty sheet. Fill in the column headings. Label the value of cell A1 as h
- ✓ Fill in cells A16 and C16 : Those cells contain the full information we have on the function. All other cells are formulae.
- ✓ Fill in cell A17
- ✓ What is the formula in C17 (remember what you've just done in 2.1) ? in B17 ?
- ✓ Select A17 to C17 and pull down (paste/copy).
- ✓ Fill in A15 to C15 and do the same thing upwards to complete the table
- ✓ Add the graph

2.4 WHAT TO HAND IN ?

On the “dropbox” area in your Math training in Chamilo, hand in a single file containing :

- ✓ The previous spreadsheet with $h = 0.1$
- ✓ The previous spreadsheet with $h = 0.5$
- ✓ The previous spreadsheet with $h = 0.01$ (you will need to insert additional lines)
- ✓ Your comments on how the value of h affects the curve
- ✓ Which one of the previous curves is likely to be the best approximation of the curve of the exponential function ?
- ✓ Give the best approximation you can of $\exp(1)$ (to 5 significant figures)