Le sujet est composé de 4 exercices indépendants. Le candidat doit traiter tous les exercices. Dans chaque exercice, le candidat peut admettre un résultat précédemment donné dans le texte pour aborder les questions suivantes, à condition de l’indiquer clairement sur la copie. Le candidat est invité à faire figurer sur la copie toute trace de recherche, même incomplète ou non fructueuse, qu’il aura développée.

Il est rappelé que la qualité de la rédaction, la clarté et la précision des raisonnements entreront pour une part importante dans l’appréciation des copies.

Avant de composer, le candidat s’assurera que le sujet comporte bien 6 pages numérotées de 1 à 6.
Exercise 1: (5 points) for all candidates

In the whole exercise, we will give rounded values to the nearest $10^{-3}$ of the probabilities requested.

**Part A** and **Part B** are independent.

**Part A**

According to a local bylaw from October 26\textsuperscript{th} 2012, the minimum authorised size of a sardine that was fished in French seas (Atlantic or Mediterranean) is 11 cm.

It is assumed that the size $T$ of the sardines can be modelled by a random variable normally distributed with expectation 13 and standard deviation 4.38.

1. Calculate the probability that a sardine fished in the French seas doesn’t have the requested size.
2. Give an interval $[T_1; T_2]$ such that approximately 95\% of the sizes of the sardines belong to that interval.

**Part B**

72\% of the supply of sardines of a fishmonger comes from a wholesaler G. The rest comes from a wholesaler H.

The wholesaler G gets 70\% of his supplies in the Mediterranean and 30\% in the Atlantic. The wholesaler H gets 55\% of his supplies in the Mediterranean and 45\% in the Atlantic.

1. Calculate the probability that a sardine sold by this fishmonger is fished in the Mediterranean and comes from the wholesaler H.
2. What is the probability that a sardine sold by this fishmonger was fished in the Mediterranean.
3. What is the probability that a sardine fished in the Atlantic and sold by this fishmonger comes from the wholesaler G?
4. The fishmonger indicates the origin of the sardines. The customers can choose Atlantic sardines or Mediterranean sardines. The fishmonger wants to adjust his supply according to the customers’ preferences. He does a market research: over a sample of 500 sold sardines, the customers chose to buy 360 Mediterranean sardines. Given that, at the moment, 65.8\% of the fishmonger’s supplies are Mediterranean sardines, is this supply consistent with the customers’ preferences?
Exercise 2: (5 points) for all candidates

We consider a ski jump which side view is given below. The chosen unit is the meter.

Let \( f \) be the function defined on the domain \([0 ; 80]\) by

\[
f(x) = 23 \cos(0.03 \, x + 0.6) + 88.
\]

We assume that the ski jump is well modelled by the graph of the function \( f \). The points A and B are the points of the curve which \( x \)-coordinates are respectively 0 and 80.

**Reminders:** Let \( g \) and \( h \) be the functions defined on \( \mathbb{R} \) by \( g(x) = \cos(ax + b) \) and \( h(x) = \sin(ax + b) \) where \( a \) and \( b \) are any real numbers. The functions \( g \) and \( h \) are differentiable on \( \mathbb{R} \) and for any real number \( x \), \( g'(x) = -a \sin(ax + b) \) and \( h'(x) = a \cos(ax + b) \).

On a given point, the slope of the ski jump is defined by the absolute value of the gradient (slope) of the tangent of the graph of \( f \) at this point.

1. Prove that the slope of the ski jump at the starting point A, rounded to the nearest \( 10^{-2} \), is 0.39.
2. Calculate the slope of the ski jump at the arrival point B. The result will be rounded at the nearest \( 10^{-2} \).
3. Calculate the \( x \)-coordinate of the point of the curve \( f \) for which the slope of the ski jump is maximum.
4. We want to cover the lateral side of the ski jump by an advertising banner. The side corresponds to the part of the figure whose limits are the graph of \( f \), the \( x \)-axis, the \( y \)-axis and the straight line \( x = 80 \). The manufacturing cost is 5 euros by m\(^2\). Rounded to the euro, how much will it cost to make this banner?
Exercise 3: (5 points) for all candidates

Before opening two new airlines, the aviation security requires that the minimum distance between two straight trajectories is at least 550 m. The purpose of the exercise is to find the minimum distance between two trajectories, in order to know whether the two airlines can be opened.

Let \((0; \vec{i}, \vec{j}, \vec{k})\) be a space orthonormal coordinate system (unit : 1 hm = 100 m). \(0\) is situated on the control tower. The two airlines trajectories are modelled by the straight lines \((D)\) and \((D')\), defined as below:

The straight line \((D)\) passes through the point \(A(-4; 6; 9)\) and the point \(L(0; 2; 10)\).

The straight line \((D')\) has for parametric representation \(\begin{cases} x = -8 + t \\ y = t \\ z = 1 \end{cases}, \ t \in \mathbb{R} \).

Let \((\Delta)\) be the straight line passing through the point \(K(-3; 5; 1)\) and of direction vector \(\vec{v} \begin{pmatrix} -1 \\ 1 \\ 8 \end{pmatrix}\).

**Part A**

1. Prove that the straight line \((\Delta)\) intercepts the straight line \((D)\) at the point \(A(-4; 6; 9)\) and that the straight line \((\Delta)\) intercepts the straight line \((D')\) at the point \(K(-3; 5; 1)\). Conclude that \((AK) = (\Delta)\).

2. Calculate the distance \(AK\).

3. Prove that the straight line \((\Delta)\) is orthogonal to the straight lines \((D)\) and \((D')\).

**Part B**

1. Using the results of **part A**, in particular the one of question 3., prove that for any point \(M\) belonging to the straight line \((D)\) and for any point \(N\) belonging to the straight line \((D')\), we have \((\overrightarrow{MA} + \overrightarrow{KN}).\overrightarrow{AK} = 0\)

2. Using the equality \(\overrightarrow{MN} = (\overrightarrow{MA} + \overrightarrow{KN}) + \overrightarrow{AK}\), prove that, for any point \(M\) belonging to the straight line \((D)\) and for any point \(N\) belonging to the straight line \((D')\), we have:
   \[ ||\overrightarrow{MN}||^2 = ||\overrightarrow{MA} + \overrightarrow{KN}||^2 + ||\overrightarrow{AK}||^2. \]

3. Deduce that for any point \(M\) belonging to the straight line \((D)\) and for any point \(N\) belonging to the straight line \((D')\), we have: \(MN^2 \geq AK^2\).

4. Is the opening of these two airlines compatible with the requirements of the aviation security?
Exercise 4:  (5 points)  This exercise is for candidates who didn’t follow the speciality course

Let \((u_n)\) and \((v_n)\) be the sequences defined by:

\[
 u_0 = 1, \quad v_0 = 0 \quad \text{and for any } n \in \mathbb{N}, \quad u_{n+1} = \frac{u_n + v_n}{2} \quad \text{and} \quad v_{n+1} = \frac{u_n - v_n}{2}.
\]

1. Copy and complete the table below and give the values of \(u_n\) and \(v_n\) for any whole number \(n\) between 1 and 3.

<table>
<thead>
<tr>
<th>(n)</th>
<th>(u_n)</th>
<th>(v_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Complete the following algorithm, so that it displays \(u_{100}\) and \(v_{100}\).

Variables:
- \(k\) is a whole number
- \(u\) is a real number
- \(v\) is a real number
- \(w\) is a real number

Initialisation:
- Assign to \(u\) the value ...
- Assign to \(v\) the value ...
- Assign to \(w\) the value 0

Processing:
- For \(k\) from ... to ...
  - Assign to \(u\) the value \(w - v\)
  - Assign to \(u\) the value \(\frac{w - v}{2}\)
- Assign to \(v\) the value ...
- End For

Output:
- Display \(u\)
- Display \(v\)

For the rest of the exercise, for any whole number, let \(z_n\) be the complex number defined by \(z_n = u_n + iv_n\).
3. Prove that for any whole number \( n \), \( z_{n+1} = \frac{1+i}{2} z_n \).

4. Prove by induction that for any whole number \( n \), \( z_n = \left( \frac{1+i}{2} \right)^n \).

5. We remind that the exponential form of the complex number \( \frac{1+i}{2} \) is \( \frac{\sqrt{2}}{2} e^{i \frac{\pi}{4}} \).

   a. Deduce that, for any whole number \( n \), we have:
   \[
   z_n = \left( \frac{\sqrt{2}}{2} \right)^n e^{i \frac{n\pi}{4}}.
   \]

   b. Deduce that, for any whole \( n \), we have:
   \[
   u_n = \left( \frac{\sqrt{2}}{2} \right)^n \cos \left( \frac{n\pi}{4} \right) \quad \text{and} \quad v_n = \left( \frac{\sqrt{2}}{2} \right)^n \sin \left( \frac{n\pi}{4} \right).
   \]

6. Prove that the sequences \( (u_n) \) and \( (v_n) \) are convergent. Give their limit.