

Activités relatives aux éléments d'Euclide – traduction d'Oliver Byrne

Activité 1 :

Citer en français les propriétés et théorèmes figurant en Annexe.

Activité 2 :

Rédiger à la manière de Byrne dans sa traduction des éléments d'Euclide, les propriétés suivantes (avec schémas et couleurs) :

- Dans un triangle la somme des mesures des angles est égale à la mesure d'un angle plat.
- Dans un quadrilatère la somme des mesures des angles est égale à la mesure de deux angles plats.
- Si un quadrilatère a trois angles droits alors le quatrième angle est droit.
- Si deux droites coupées par une troisième définissent des angles alternes-internes égaux, alors ces deux droites sont parallèles.

(certaines des propriétés figurant en annexe, notamment celle sur les angles opposés par le sommet peuvent être données aux élèves dans le cadre de l'activité 2
certaines des propriétés de l'activité 2 peuvent être demandées aux élèves en démonstrations, à la manière de Byrne)





Activité 3 :

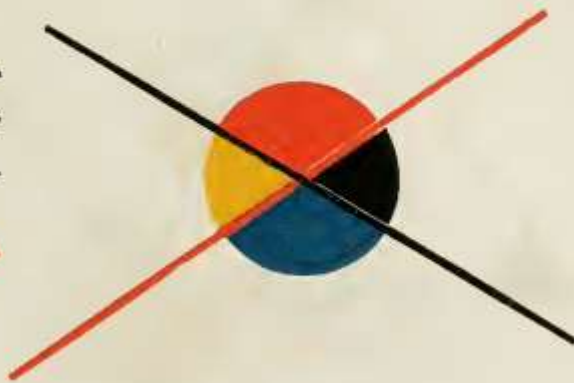
Réécrire en langage mathématique actuel les démonstrations des propriétés et théorèmes figurant en Annexe

Annexe :



If two right lines (— and —) intersect one another, the vertical an-

gles  and , 
and  are equal.



$$\text{yellow sector} + \text{red sector} = \text{semicircle}$$

$$\text{black sector} + \text{red sector} = \text{semicircle}$$

$$\therefore \text{yellow sector} = \text{black sector}$$

In the same manner it may be shown that

$$\text{red sector} = \text{blue sector}$$

Q. E. D.





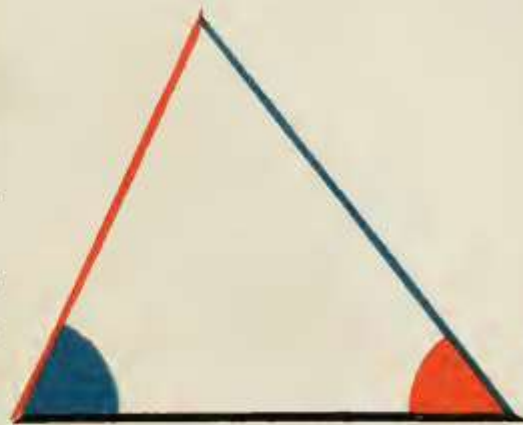
*I*n any triangle







one angle  be greater

than another  the side

 which is opposite to the greater angle, is greater than the side  opposite the less.





If  be not greater than  then must

 = or \supset .

If  =  then


 =  (pr. 5.);

which is contrary to the hypothesis.

 is not less than ; for if it were,

 \supset  (pr. 18.)

which is contrary to the hypothesis:

\therefore  \supset .

Q. E. D.



IF two triangles have two sides of the one respectively equal to two sides of the other, (— to — and — to —) and the angles (▲ and ▲) contained by those equal

sides also equal; then their bases or their sides (— and —) are also equal: and the remaining and their remaining angles opposite to equal sides are respectively equal (▲ = ▲ and ▲ = ▲): and the triangles are equal in every respect.

Let the two triangles be conceived, to be so placed, that the vertex of the one of the equal angles, ▲ or ▲ ; shall fall upon that of the other, and — to coincide with —, then will — coincide with — if applied: consequently — will coincide with —, or two straight lines will enclose a space, which is impossible (ax. 10), therefore — = —, ▲ = ▲

and ▲ = ▲, and as the triangles ▲ and ▲ coincide, when applied, they are equal in every respect.

Q. E. D.



IF a straight line be divided into any two parts — —, the square of the whole line is equal to the squares of the parts, together with twice the rectangle contained by the parts.

$$—^2 = —^2 + —^2 + \text{twice } — \cdot —$$

Describe (pr. 46, B. 1.)
 draw (post. 1.),

and { } (pr. 31, B. 1.)

$$\triangle = \triangle \text{ (pr. 5, B. 1.)}$$

$$\triangle = \triangle \text{ (pr. 29, B. 1.)}$$

$$\therefore \triangle = \triangle$$

\therefore by (prs. 6, 29, 34. B. 1.) is a square = $—^2$.

For the same reasons is a square = $—^2$,


$$\square = \square = — \cdot — \text{ (pr. 43, b. 1.)}$$

but = + + + ,

$$\therefore —^2 = —^2 + —^2 + \text{twice } — \cdot —$$


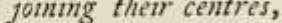
Q. E. D.

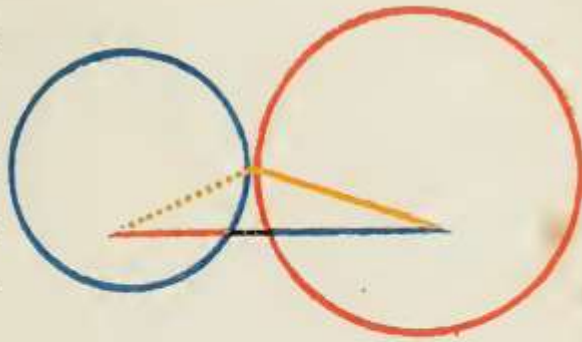





Two circles  and



touch one ano-

ther externally, the straight line  joining their centres,  passes through the point of contact.







If it be possible, let  join the centres, and not pass through a point of contact; then from a point of contact draw  and  to the centres.

Because  +  \square  

(B. 1. pr. 20.),

and  =  (B. 1. def. 15.),

and  =  (B. 1. def. 15.),

\therefore  +  \square  , a part greater than the whole, which is absurd.

The centres are not therefore so placed, that the line joining them can pass through any point but the point of contact.

Q. E. D.